UNIT - I : Electrostatics

Chapter - 1 : Electric Charges and Fields

TOPIC-1

Electric Field and Dipole

Quick Review

Electric Charge

- Electric Charge is that property of a matter due to which, similar charges repel each other and an attraction force takes place among opposite charges.
- Point Charge is an accumulation of electric charges at a point.
- Electrons are smallest and lightest particles in an atom having negative charge as these are surrounded by invisible force field known as electrostatic field.
- Protons are larger and heavier than electrons with positive electrical charge which is similar in strength as electrostatic field in an electron with opposite polarity.
- Two electrons and protons will tend to repel each other as they have negative and positive electrical charges.

$$F \longleftarrow \overset{e^-}{\bullet} \qquad \overset{e^-}{\bullet} \longrightarrow F$$

$$F \longleftarrow \overset{p}{\bullet} \qquad \overset{p}{\bullet} \longrightarrow F$$

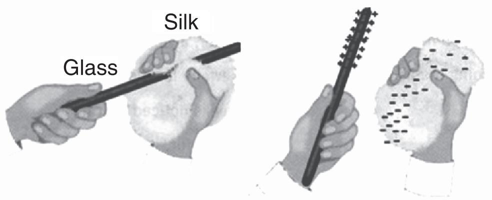
- The electrons and protons will get attracted towards each other due to their unlike charges.
- The charge present on the electron is equal and opposite to charge on the proton.

Electric Charge

- Electrostatic charge means the charge is at rest.
- Electrostatic charge is a fundamental quantity like length, mass, and time.
- Charge on body is expressed as $q = \pm ne$
- The magnitude of charge is independent of the speed of particle.
- Based on the flow of charge across them, materials are divided as :
  - conductors - allow electric charge to flow freely - metals
  - semi-conductors - behave as conductor or insulator - silicon
  - insulators - do not allow electric charge to flow freely - rubber, wood, plastic
- Net charge is given by :
  - Charging by friction -charging insulators
  - Charging by conduction -charging metals and other conductors
  - Charging by induction -charging metals and other conductors

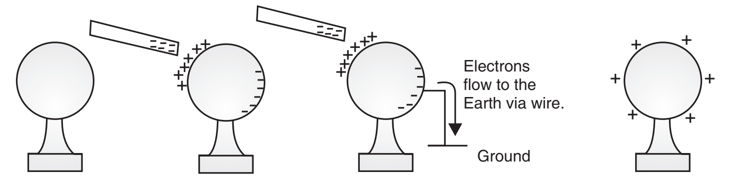
Charging by Induction

- Charging by induction means charging without contact
- On rubbing a glass rod and silk cloth piece together, glass rod get positive charge whereas silk cloth gets negative charge.



- If a plastic rod is rubbed with wool, it becomes negatively charged.
- If a negatively charged rod is brought near neutral metal with insulator mounting, it repels free electrons and rod attracts positive charges on metal.

- If far end is connected to Earth by a wire, electrons will flow towards ground while positive charges are kept captive by the rod.



Neutral sphere | A negative rod nearby separates (+)s & (-)s. | With rod nearby the negative end may be discharged. | With rod away, the (+)s distribute evenly

Properties of Electric Charge

Addition of charges

- If a system contains three point charges q_1, q_2 and q_3, then the total charge of the system will be the addition of q_1, q_2 and q_3, *i.e.*, charges will add up.

$$Q = q_1 + q_2 + q_3$$

Conservation of charges

- Electric charge is always conserved. It is the sum of positive and negative charges present in an isolated system, which remains constant.
- Charge can be created and destroyed, but only in positive-negative pairs.

Quantization of charges

- Electric charge is always quantized *i.e.*, electric charge is always an integral multiple of charge 'e'.
- Net charge q_{net} of an object having N_e electrons, N_p protons and N_n neutrons is:
- $q_{net} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = ne$
- It is the property through which any charge exists in discrete packets of minimum charges where positive and negative charge is denoted as q in equation $q = ne$, where n is an integer, n = number of electrons present on the body and e = charge on an electron
- Neutron (n) : $m = 1.675 \times 10^{-27}$ kg; $q = 0$
- Proton (p) : $m = 1.673 \times 10^{-27}$ kg; $q = +1.602 \times 10^{-19}\ C$
- Electron (e): $m = 9.11 \times 10^{-31}$ kg; $q = -1.602 \times 10^{-19}\ C$

Coulomb's Law

- The force of attraction or repulsion between two point charges q_1 and q_2 separated by a distance r is directly proportional to product of magnitude of charges and inversely proportional to square of distance between charges, written as :

$$F = k\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

Like charges repel Unlike charges attract

Where,
F = Force of attraction/repulsion between charges q_1, q_2.
q_1, q_2= Magnitudes of charge 1 and charge 2
r = Distance between charges q_1, q_2
k = Constant whose value depends on medium where charges are kept.

$$k = \frac{1}{4\pi\varepsilon_0}$$

As $\varepsilon = \varepsilon_0\varepsilon_r$ $k = \dfrac{\varepsilon_r}{4\pi\varepsilon}$

ε_0 = permittivity of vacuum = 8.854×10^{-12} F/m
ε_r = relative permittivity of medium with respect to free space.

- For vacuum, relative permittivity, $\varepsilon_r = 1$,
- As $\varepsilon = \varepsilon_0$, force of attraction/repulsion among two electric charges q_1, q_2 placed in vacuum and medium is :

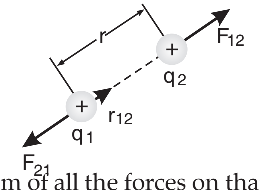
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ (vacuum)}$$

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2} \text{ (medium)}$$

- The unit coulomb (C) is derived from the SI unit ampere (A) of the electric current.
- Current is the rate $\dfrac{dq}{dt}$ at which charge moves past a point or through a region $i = \dfrac{dq}{dt}$, hence 1C=(1A) × (1s)

- The vector form of Coulomb force with $\vec{r}_{12}$ = unit vector from $\vec{q}_1$ to $\vec{q}_2$ is given as:

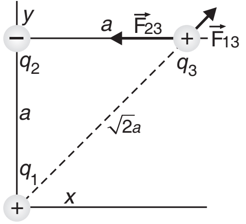
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{q}_1\vec{q}_2}{\vec{r}_{21}^2}\hat{r}_{21} \text{ and } \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{q}_1\vec{q}_2}{\vec{r}_{12}^2}\hat{r}_{12}$$



$$\Rightarrow \qquad \vec{F}_{21} = -\vec{F}_{12}$$

Principle of Superposition

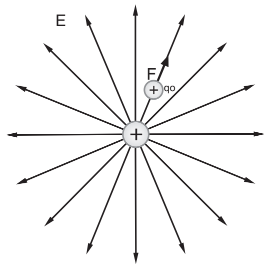
- The force on any charge due to a number of other charges at rest is the vector sum of all the forces on that charge due to the other charges, taken one at a time.
- The individual forces are unaffected due to presence of other charges.

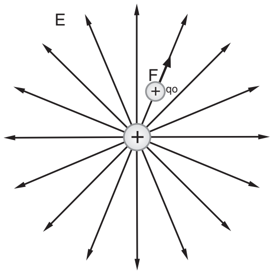


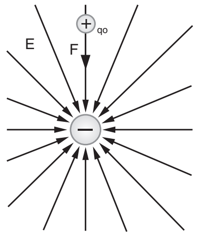
- force exerted by $\vec{q}_1$ on $\vec{q}_3$ = $\vec{q}_3 = \vec{F}_{13}$
- force exerted by $\vec{q}_2$ on $\vec{q}_3$ = $\vec{F}_{23}$

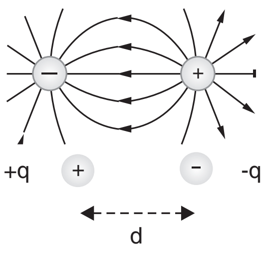
‹ net force exerted on q_3 is vector sum of $\vec{F}_{13}$ and $\vec{F}_{23}$

Electric field









- The space around a charge up to which its electric force can be experienced
- is called electric field.
- If a test charge q_0 is placed at a point where electric field is E, then force on the test charge is $F = q_0E$
- The electric field strength due to a point source charge 'q' at an observation point 'A' at a distance 'r' from the source charge is given by $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|A-r|^3}(A-r)$ (point charge)
- The unit of electric field is N/C
- Electric field inside the cavity of a charged conductor is zero.
- If a charged/uncharged conductor is placed in external field, the field in conductor is zero.
- In case of charged conductor, electric field is independent of the shape of conductor.

Electric field lines

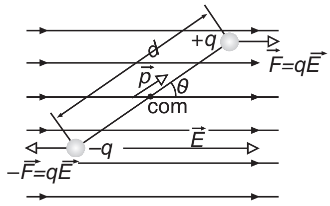
- Electric field lines are imaginary lines that extend from positive charge towards negative charge.
- Direction of electric field lines around positive charge is imagined by positive test charge q_0 located at points around source charge.
- Electric field has same direction as force on positive test charge.
- Electric field lines linked with negative charge are directed inward described by force on positive test charge q_0.
- Field lines never cross each other.
- Strength of field is encoded in density of field lines.

Electric Dipole

- Dipoles make fields
- The system formed by two equal and opposite charges separated by a small distance is called an electric dipole.
- The force on a dipole in an electric field is zero both in a stable as well as in an unstable equilibrium.
- The potential energy of a dipole in an electric field is minimum in a stable equilibrium and maximum in an unstable equilibrium.

Moment of Force

- In a dipole, when net force on dipole from field is zero and center of mass of dipole remains fixed, then forces on charged ends produces net torque τ about its center of mass (com).



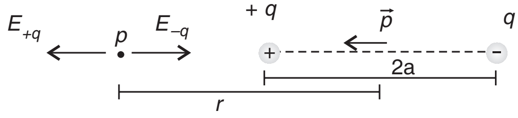
$$\tau = F\,d\sin\theta = qE(d)\sin\theta = pE\sin\theta$$
$$\vec{\tau} = \vec{p}\times\vec{E}$$

- If $\theta = 0^{\circ}$, 180° or 360°, dipole exists in equilibrium state.
- If $\theta = 0^{\circ}$ or 360°, dipole exists in stable state.
- If $\theta = 180^{\circ}$, dipole exists in unstable state.

- In uniform electric field, dipole experiences torque, net force on dipole is zero.
- In uniform electric field, dipole experiences rotatory motion.
- In non-uniform electric field, dipole experiences torque and net force.
- In non-uniform electric field, dipole experiences rotatory and translatory motion.
- In this, Torque aligns dipole with electric field and becomes zero.
- In this, Torque direction is normal to the plane going inward.

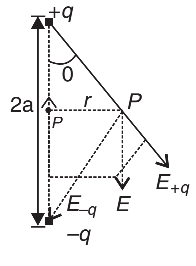
Electric Dipole Moment

- Dipole moment is a vector quantity whose unit is coulomb-meter (Cm).
- The maximum value of electric field is at surface of shell of charge since cell is cloud of charge.
- Dipole moment vector $\vec{p}$ of electric dipole is $\vec{p} = \vec{q}\times 2a$ between pair of charges q, $-q$ along the line



- For point P at distance r from centre of dipole on charge q, for $r>>a$, total field at point P is

$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$



- For point P on the equatorial plane due to charges $+q$ and $-q$, electric field of dipole at large distances

$$E = \frac{-p}{4\pi\varepsilon_0 r^3}$$

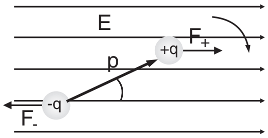
Know the Terms

- **1 coulomb :** When two point charges are placed at a distance of 1 m in vacuum and repel each other with force of 9×10^9 N, the charge on each sphere is known as 1 coulomb.
- **Flux :** The rate of flow through an area or volume. Also it is the product of an area and vector field across the area.
- **Electric Flux :** Rate of flow of an electric field through an area or volume represented by number of E field lines penetrating a surface.
- **Electric line of force :** It is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.
- **Symmetry :** The balanced structure of an object, the halves of which are alike.

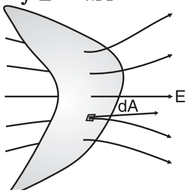
Know the Formulae

- **Coulomb's force :** $F_{12} = \dfrac{k|q_1q_2|}{r^2_{12}}$;

  where all alphabets are in their usual meanings.
- **Electric field due to point charge q :** $E = \dfrac{k|q|}{r^2}$
- **Electric field due to a dipole at a point on the dipole axis :** $E = \dfrac{2p}{4\pi\varepsilon_0 r^3}$ $(r >>> a)$
- Electric field at a point on equitorial plane $E = \dfrac{-p}{4\pi\varepsilon_0 r^3}$ $(r >>> a)$



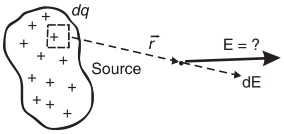
- **Potential energy for dipole :** U = – pEcos θ
- **Flux through non uniform electric field :** $\phi = \int E \times dA$



- **Field of continuous charge distribution :**

$$E_x = \int \frac{k_\varepsilon \times dq}{r^2} \cos\theta$$

$$E_y = \int \frac{k_\varepsilon \times dq}{r^2} \sin\theta$$



- **Gauss' Law :** $\phi = \int E \cdot dA = \dfrac{q_{enclose}}{\varepsilon_0}$
- **Electric field due to long line of charge :** $E_{line} = \dfrac{\lambda}{2\pi\varepsilon_0 r}$, where λ = linear charge density
- **Electric field due to infinite sheet of charge :** $E_{sheet} = \dfrac{\sigma}{2\varepsilon_0}$, where σ = surface charge density

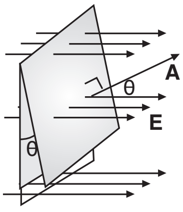
TOPIC-2

Gauss's Theorem and Its Applications

Quick Review

Electric Flux

- Electric flux is proportional to algebraic number of lines leaving the surface, outgoing lines with positive sign, incoming lines with negative sign.

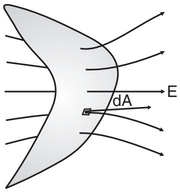


- Due to arbitrarily arrangement of electric field lines, electric flux can be quantify as $\phi_E = EA$
- If vector A is perpendicular to surface, amount of vector A parallel to electric field is $A\cos\theta$

  $A_{II} = A\cos\theta$

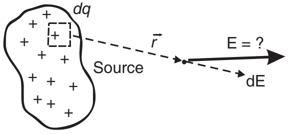
  $\phi_E = EA_{II} = EA\cos\theta$

- In non-uniform electric field, the flux will be $\phi_E = \int E dA$



Continuous Charge Distribution

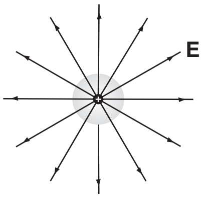
- The continuous charge distribution system is a system in which the charge is uniformly distributed over the conductor.
- Continuous charge distribution can be linear, surface, and volume charge distribution.



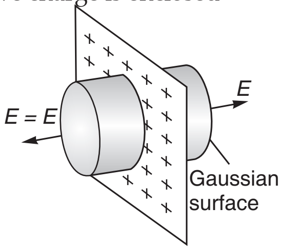
- Field of continuous charge distribution
  - $E_x = \int \frac{k_s \cos\theta}{r^2} dq$
  - $E_y = \int \frac{k_s \sin\theta}{r^2} dq$

Gauss law

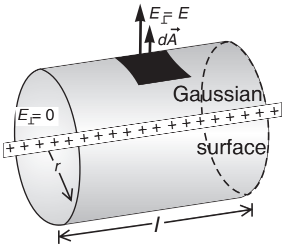
- Gauss law is addition of sources in a closed surface that will be equal to total flux through the surface.



- The electric field at all points on Gaussian surface is $\phi = E\int dA = \frac{q}{\varepsilon_0}$
- If there is a positive net flux, net positive charge is enclosed
- If there is a negative net flux, net negative charge is enclosed



- If there is zero net flux, no net charge is enclosed
- The equation for point charge on Gaussian surface is $E = \frac{q}{4\pi r^2 \varepsilon_0}$
- In an insulating sheet, charge remains in the sheet, so electric field $E = \frac{\sigma}{2\varepsilon_0}$



- Gauss' Law works in cases of cylindrical, spherical and rectangular symmetries.
- The field outside the wire points radially outward which depend on distance from wire $E = \frac{\lambda}{2\pi r \varepsilon_0}$, where λ is linear density of charge

Know the Terms

- **Closed surface :** It is a surface which divides the space in inside and outside region, where one can't move from one region to another without crossing the surface
- **Gaussian surface :** It is a hypothetical closed surface having similar symmetry as problem on which we are working.
- **Relative permittivity (ε_r) :** It is defined as ratio of Coulomb's force F between two point charges which is placed in free space to Coulomb's force F' between the same charges having same distance of separation when kept in a medium *i.e.*, $E_r = \frac{F}{F'}$
- **Dielectric constant (K) :** It is the ratio of field without dielectric (E_0) to the net field (E) with dielectric given as $k = \frac{E_0}{E}$, where $E \le E_0$. Dielectric constant is ≥ 1. The larger the dielectric constant more will be the charge stored.
- **Electrostatic Shielding :** It is the phenomenon of protecting certain region of space from external electric field.
- **Dielectric :** The non-conducting material in which charges are easily produced on the application of electric field is called dielectric. *e*.g. Air, H_2 gas, glass, mica, paraffin wax, transformer oil etc.

Know the Formulae

- **Electric flux through an area A :** $\phi = E.A = EA\cos\theta$
- **Electric flux through a Gaussian surface :** $\phi = \int E.dS$
- **Gauss's Law :** $\phi = \frac{q_{enc}}{\varepsilon_0}$
- **Electric Field due to an infinite line of charge :** $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$
- where, E = electric field [N/C]

  λ = charge per unit length [C/m]

  ε_0 = permittivity of free space = 8.85×10^{-12} [C^2/Nm^2]

  r = distance (m)

  $k = 9 \times 10^9$ Nm^{2}C^{-2}
- Electric field due to a ring at a distance x is $E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(r^2 + x^2\right)^{3/2}}$
- When, $x >>> r$; $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$
- When $x <<< r$; $E = 0$
- Electric field due to a disc charge $E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2 + x^2}}\right]$

  where ,

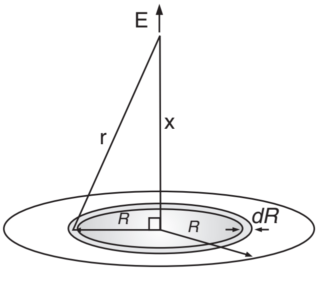
  E = electric field [N/C]

  σ = charge per unit area [C/m^2]

  $\varepsilon_0 = 8.85 \times 10^{-12}$[C^2/Nm^2]

  x = distance to charge [m]

  R = radius of the disc [m]



- Electric field due to a thin infinite sheet $E = \dfrac{\sigma}{2\varepsilon_0}$
- Electric field inside a spherical shell $E = 0$
- Electric field outside a spherical shell $E = \dfrac{kq}{r^2}$ for $r > R$

  where,

  r = distance from the centre of sphere to the charge in *m*.

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Chapter - 2 : Electrostatic Potential and Capacitance

TOPIC-1
Electric Potential

Quick Review

Electric potential

- Electric potential is the amount of work done by an external force in moving a unit positive charge from one point to another in electrostatic field.
- It is written as $V = \dfrac{W}{q}$

  where, W = work done in moving charge q through the field (J), q = charge being moved through the field (C)
- The SI unit of electric potential $\dfrac{J}{C}$, Volt, $\dfrac{Nm}{C}$
- Also we can write $V = \dfrac{1}{4\pi\varepsilon}\dfrac{q}{r}$

  where, q = charge causing the field, ε = permittivity, r = separation between centre of charge and point

Potential difference

- Electric potential difference is the difference in electric potential (V) between the final and the initial location, when work is done upon a charge to change its potential energy.

  Electric potential difference = $\dfrac{\text{Work}}{\text{Charge}} = \dfrac{\Delta PE}{\text{Charge}}$

  Between two points A and B, $W_{AB} = -V_{AB} \times q$

  Where, $V_{AB} = V_B - V_A$ is potential difference between A and B.
- In a region of space having an electric field, the work done by electric field dW, when positive point charge q, is displaced by ds, then,

  $dW = q\vec{E} \times ds$

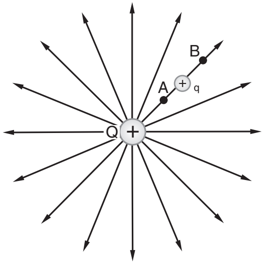
  $$\Delta V = V_{AB} = V_B - V_A = -\frac{W_{AB}}{q} = -\frac{\int_A^B qE \times ds}{q} = -\int_A^B E \times ds$$

Electric potential due to point charge

- The electric potential by point charge q, at a distance r from the charge, can be shown as $V_E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$

  Where, ε_0 is dielectric constant (permittivity of vacuum)
- Electric potential due to dipole at a point at distance r and making an angle θ with the dipole moment p is given by, $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p\cos\theta}{r^2}$

- Electric potential is a scalar quantity.
- Dimension of Electric potential is $[M^1L^2T^{-3}A^{-1}]$



- For a single point charge q the potential difference between A and B is given by,

$$\Delta V = V_B - V_A = -\int_A^B E \times ds = -\int_A^B Eds\cos 0 = -\int_A^B E \times ds$$

where, E is the field due to a point charge, $ds = dr$, so that,

$$V_B - V_A = -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

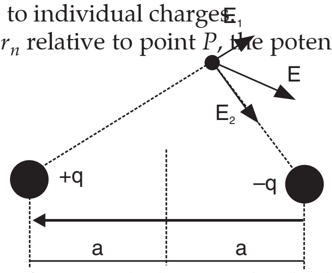
- If $r_B = \infty$, then $V_B = 0$ so,

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_A} = \frac{kq}{r_A}$$

Dipole and system of charges

- The net potential due to a dipole at any point on its equatorial line is always zero. So work done in moving a charge on equatorial line is always zero.
- Electric dipole is two charged objects having equal but opposite electric charges which are separated by a distance.
- Potential at a point due to system of charges is the sum of potentials due to individual charges.
- In a system of charges $q_1, q_2, q_3, \ldots q_n$ having positive vectors $r_1, r_2, r_3, \ldots.. r_n$ relative to point P, the potential V_1 at point P due to charge q_1 will be

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$



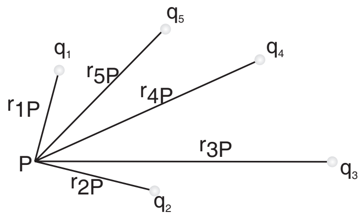
- Also, potential at P due to charge q_2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

- The potential at point P due to total charge configuration is algebraic sum of potentials due to individual charges, so, $V = V_1 + V_2 + V_3 + \ldots.. + V_n$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots. + \frac{q_n}{r_n}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$



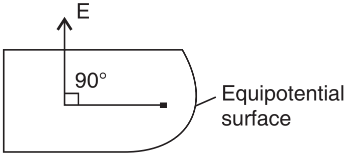
- It is known that in a uniformly charged spherical shell, electric field outside the shell with outside potential is given as :

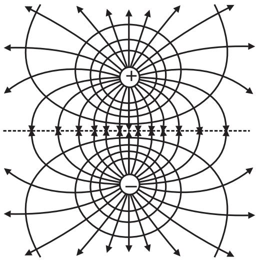
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}(r \geq R)$$

where, q is total charge on shell and R being shell radius.

Equipotential surfaces

- Equipotential Surface is a surface in space on which all points have same potential. It requires no work to move the charge on such surface, hence the surface will have no E component, so E will be at right angles to the surface
- Work done in moving a charge over equipotential surface is zero.
- Electric field is always perpendicular to equipotential surface.





- Spacing among equipotential surfaces allows to locate regions of strong and weak fields.
- Equipotential surfaces never intersect each other.
- The intersecting point of two equipotential surfaces results in two values of electric potential which is impossible.

Electric potential energy of system of two point charges

- Work done is path independent.
- Electric potential energy of system charges is work done in bringing the charges from infinity near each other for obtaining a system.
- In two point charges system having charges q_1 and q_2 separated by distance *r*, two or more electric charges will either attract or repel each other.

  Electric potential when q_1 is placed (as shown in the figure):

  $$V(\vec{r}) = V_2 = k\frac{q_1}{r_{12}}$$

  Electric potential energy when q_2 is placed in potential V_2:

  $$U = q_2V_2 = k\frac{q_1q_2}{r_{12}}$$

  Electric potential when q_2 is placed :

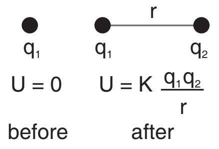
  $$V(\vec{r}) = V_1 = k\frac{q_2}{r_{12}}$$

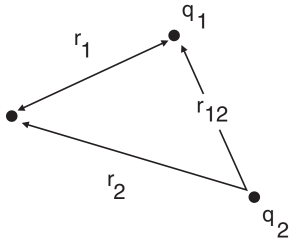
  Electric potential energy when q_1 is placed in potential V_1 :

  $$U = q_1V_1 = k\frac{q_1q_2}{r_{12}}$$

  Electric potential energy of q_1 and q_2:

  $$U = \frac{1}{2}\sum_{i=1}^{2} q_iV_i$$





- Field and potential due to spherical shell

  (i) On the Surface :

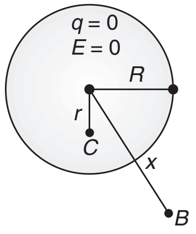
  $$E_A = \frac{Kq}{R^2},\ V_A = \frac{Kq}{R}$$

  (ii) Outside the Surface :

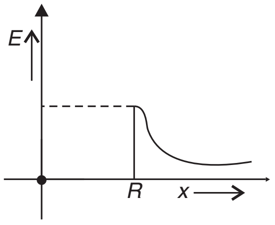
  $$E_B = \frac{Kq}{x^2},\ V_B = \frac{Kq}{x}$$

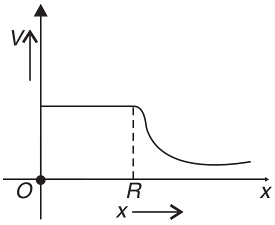
  (iii) Inside the Sphere

  $$E_C = 0,\ V_C = \frac{Kq}{R}$$



  Field Graph :





Electric dipole in an electrostatic field

- Electric potential due to a dipole at a point at distance *r* and making an angle θ with the dipole moment *p* is given by

  $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Know the Terms

- **Electric Potential difference :** The electric potential difference between two points in an electric field is defined as the amount of work done per unit positive test charge in moving the test charge from one point to the other against electrostatic force due to field.

- **Equipotential surface :** The surface at every point of which, the electric potential is same.
- **Electrostatic Potential energy of a system of charges :** It is defined as the work done to put the charges constituting the system at their respective locations after having been removed to infinity.
- **Dielectrics :** Non-conducting substances with no charge carriers.
- **Electrostatic shielding :** It is a field inside cavity of a conductor which is always zero that remains shielded from external electric effects.

Know the Formulae

- Electric Potential $(V) = \frac{W}{q}$, measured in volt; 1 volt = 1 Joule / coulomb.
- Electric potential difference or "voltage" $(\Delta V) = V_f - V_i = \frac{\Delta U}{q} = \frac{W}{q}$

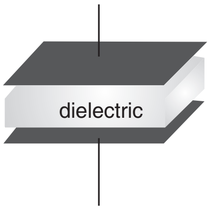
  where, U = electric potential energy,
  W = work done by the electric field.
- Electric potential due to a point charge q at a distance r away : $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
- Finding V from E : $V_f - V_i = -\int_i^f \vec{E}.d\vec{S}$
- Potential energy of two point charges in absence of external electric field : $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}}\right]$
- Potential energy of two point charges in presence of external electric field : $q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

  where,
  q_1, q_2 = point charges
  V = potential at positions of q_1, q_2
  r_{12} = distance between charges

TOPIC-2
Capacitance

Quick Review

Conductors and insulators



- Conductors are the materials through which charge can move freely.
  Examples : metals, semi-metals as carbon, graphite, antimony and arsenic.
- Insulators are materials in which the electrical current will not flow easily. Such materials cannot be grounded and do not easily transfer electrons. Examples plastics and glass.

Dielectrics

- These are the material in which induced dipole moment is linearly proportional to applied electric field.
- Electrical displacement or electrical flux density $D = \varepsilon_r\varepsilon_0 E$.
  Where, ε_r= relative permittivity, ε_0 = permittivity of free space and E is electric field.
- If a dielectric is kept in between the plates of capacitor, capacitance increases by factor 'κ' (kappa) known as dielectric constant, so $C = \kappa\varepsilon_0\frac{A}{d}$

  Where,
  κ = dielectric constant of material also called relative per symbol mittivity.

$$\kappa = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

  ε_0 = permittivity of free space, A = area

Material	κ	Dielectric strength
Air	1.00059	3
Paper	3.7	16
Glass	4-6	9
Water	80	-

- In dielectric, polarisation and production of induced charge takes place when dielectric is kept in an external electric field.

Electric polarization

- Electric polarization P is the difference between electric fields D (induced) and E (imposed) in dielectric due to bound and free charges written as $P = \dfrac{D-E}{4\pi}$
- In terms of electric susceptibility : $P = \chi_e E$
- In MKS : $P = \varepsilon_0 \chi_e E$, where ε_0 is permittivity of free space
- The dielectric constant K is always greater than 1 as $\chi_e > 0$

Capacitor

- A capacitor is a device which is used to store charge.
- Amount of charge 'Q' stored by the capacitor depends on voltage applied and size of capacitor.
- Capacitor consists of two similar conducting plates placed in front of each other where one plate is connected to positive terminal while other plate connected to negative terminal.
- In this, electric charge stored between plates of capacitor which is directly proportional to potential difference between plates, *i.e.*,
  $Q = CV$
  where, C = Capacitance of capacitor, V= potential difference between the plates
- In capacitor, energy is stored in form of electric field, in the space between the plates.

Capacitance

- Capacitance of a capacitor is ratio of magnitude of charge stored on the plate to potential difference between the plates, written as $C = \dfrac{Q}{\Delta V}$
  where,
  C = capacitance in farads (F), Q = charge in Coulombs (C)
  ΔV = electric potential difference in Volts (V),
- SI unit of capacitance is farad (F)
- $1\ F = \dfrac{1\ C}{1\ V} = 9 \times 10^{11}$ stat farad where stat-farad is electrostatic unit of capacitance in C.G.S. system
- Capacitance of a conductor depends on size, shape, medium and other conductors in surrounding.
- Parallel plate capacitor with dielectric among its plates has capacitance given as $C = k\varepsilon_0 \dfrac{A}{d}$, where ε_0 is permittivity of free space with SI units = 8.85×10^{-12} F/m
- Capacitor having capacitance of 1 Farad is too large for electronics applications, so components with lesser values of capacitance such as μ (micro), n (nano) and p (pico) are applied such as :

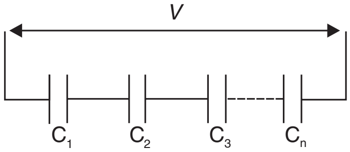
PREFIX	MULTIPLIER	
μ	10^{-6} (millionth)	1000000 μF = 1F
N	10^{-9} (thousand-millionth)	1000 nF = 1μF
P	10^{-12} (million-millionth)	1000 pF = 1nF

Combination of capacitors in series and parallel

Capacitors in series

- **(i)** If a number of capacitors of capacitances C_1, C_2, C_3, ... C_n are connected in series, then their equivalent capacitance is given by :

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots\ldots\ldots \frac{1}{C_n}$$



- In series combination, the charge on each capacitor is same, but the potential difference on each capacitor depends on their respective capacitance, *i.e.*,

$$q_1 = q_2 = q_3 \ldots\ldots\ldots\ldots q_n = q$$

- If $V_1, V_2, V_3, \ldots\ldots, V_n$ be the potential differences across the capacitors and V be the emf of the charging battery, then

$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots + V_n$$

- As charge on each capacitor is same, therefore

$$q = V_1C_1 = V_2C_2 = V_3C_3 \ldots\ldots\ldots$$

  the potential difference is inversely proportional to the capacitance, i.e.

$$V \propto \frac{1}{C}$$

- In series, potential difference across largest capacitance is minimum.
- The equivalent capacitance in series combination is less than the smallest capacitance in combination.

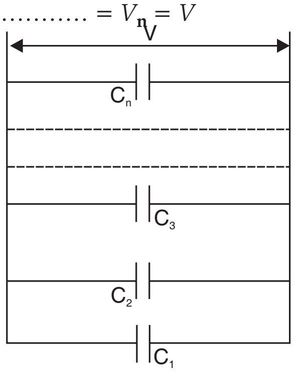
Capacitors in parallel

- (i) If a number of capacitors of capacitances $C_1, C_2, C_3 \ldots\ldots\ldots C_n$ are connected in parallel, then their equivalent capacitance is given by,

$$C_p = C_1 + C_2 + C_3 + \ldots\ldots\ldots + C_n$$

- In parallel combination, the potential difference across each capacitor is same and equal to the emf of the charging battery, *i.e.*,

$$V_1 = V_2 = V_3 = \ldots\ldots\ldots\ldots = V_n = V$$



  while the charge on different capacitors may be different.

- If $q_1, q_2, q_3, \ldots\ldots\ldots, q_n$ be the charges on the different capacitors, then

$$q_1 + q_2 + q_3 + \ldots\ldots\ldots\ldots + q_n = VC_p$$

- As potential drop across each capacitor is same, so $V = q_1C_1 = q_2C_2 = q_3C_3 = \ldots. = q_n C_n$

$$\Rightarrow \qquad V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \ldots\ldots = \frac{q_n}{C_n}$$

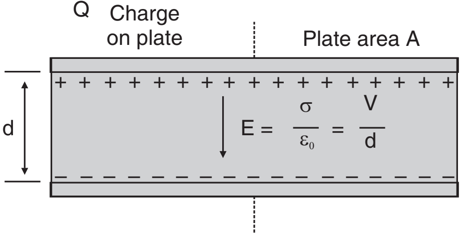
- The charges on capacitors are directly proportional to capacitances, *i.e.*, $q \propto C$
- Parallel combination is useful when large capacitance with large charge gets accumulated on combination.
- Force of attraction between parallel plate capacitor will be $F = \frac{1}{2}\left[\frac{QV}{d}\right] = \frac{1}{2}QE$ where Q is charge on capacitor.

Capacitance of parallel plate capacitor with and without dielectric medium between the plates

- Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance where space between them is filled by dielectric medium
- The electric field between two large parallel plates is given as:

  $E = \frac{\sigma}{\varepsilon}$, where, σ = charge density and ε = permittivity

  $\sigma = \frac{Q}{A}$, where, Q = charge on plate and A = plate area



- Capacitance of parallel-plate capacitor with area A separated by a distance d is written as $C = \varepsilon_r \varepsilon_0 \frac{A}{d}$

  where,

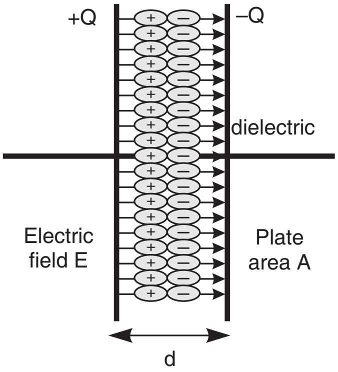
  C is capacitance;

A is area of overlap of two plates;

ε_r is relative static permittivity or dielectric constant of material between the plates;

ε_0 is electric constant 8.854×10^{-12} F m^{-1};

d is separation distance between the plates.



- If a dielectric slab is placed in between the plates of capacitor, then its capacitance will increase by certain amount.
- Capacitance of parallel plate capacitor depends on plate area *A*, distance *d* between the plates, medium between the plates (*K*) and not on charge on the plates or potential difference among the plates.
- If we have number of dielectric slabs of same area as the plates of the capacitor and thicknesses t_1, t_2, t_3,.... and dielectric constant K_1, K_2, K_3...... between the plates, then the capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + ...}, \text{ where, } d = t_1 + t_2 + t_3 + ...$$

- If slab of conductor of thickness *t* is introduced between the plates, then

$$C = \frac{\varepsilon_0 A}{\frac{t}{K} + \frac{(d-t)}{1}} = \frac{\varepsilon_0 A}{\frac{t}{\infty} + \frac{(d-t)}{1}} \qquad \text{(Here, } K = \infty \text{ for a conductor)}$$

$$C = \frac{\varepsilon_0 A}{d-t}$$

- When the medium between the plates consists of slabs of same thickness but areas A_1, A_2, A_3,... and dielectric constants K_1, K_2, K_3..., then capacitance is given by

$$C = \frac{\varepsilon_0 (K_1 A_1 + K_2 A_2 + K_3 A_3 ....)}{d}$$

$$\therefore \qquad K = \frac{C_m}{C_0} = \frac{\text{capacitance in medium}}{\text{capacitance in vacuum}}$$

- When space between the plates is partly filled with medium of thickness *t* and dielectric constant *K*, then capacitance will be :

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

where : ε_0 = electric constant, *d* = separation distance between the plates

A = plate area, *K* = dielectric constant of material between the plates

When there is no medium between the plates, then *K* =1, so

$$C_{\text{vacuum}} = \frac{\varepsilon_0 A}{d}$$

where : ε_0 = electric constant, *d* = separation distance between the plates

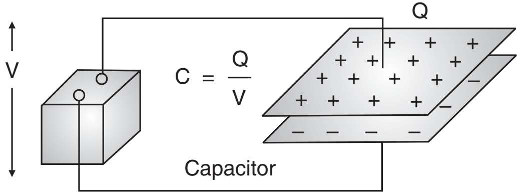
- Capacitance of spherical conductor of radius *R* in a medium of dielectric constant *K* is given by $C = 4\pi\varepsilon_0 KR$

Energy stored in capacitor

- In capacitor, energy gets stored when a work is done on moving a positive charge from negative conductor to positive conductor against the repulsive forces.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

where, U = energy stored, C = capacitance, V = potential difference



Know the Terms

- **Capacitor :** Electronic component that can store electric charge.
- **Polar atom :** Atom in which positive and negative charges possess asymmetric charge distribution about its centre.
- **Polarisation :** The stretching of atoms of a dielectric slab under an applied electric field.
- **Dielectric strength :** The maximum value of electric field that can be applied to dielectric without its electric breakdown.
- **Capacitance :** It is the amount of charge that gets stored per unit volt.
- **Dielectric :** It is an electrically insulated or non-conducting material considered for its electric susceptibility.
- **Parallel plate capacitor :** Two identical conducting plates which are separated by a distance.

Permittivity :

It is a property of a dielectric medium that shows the forces which electric charges placed in medium exerts on each other.

OR

It is the measure of resistance that is encountered when forming an electric field in a particular medium. More specifically, permittivity describes the amount of charge needed to generate one unit of electric flux in a porticular medium

Know the Formulae

- Capacitance, $C = \frac{Q}{V}$, measured in Farad; 1F = 1 coulomb/volt
- **Parallel plate capacitor :**

$$C = K\varepsilon_0 \frac{A}{d}$$

where,
C = capacitance [farads (F)]
K = dielectric constant
ε_0 = permittivity of free space = 8.85×10^{-12} C^2/Nm^2
A = area of one plate [m^2]
d = separation between the plates [m]

- **Cylindrical capacitor :**

$$C = 2\pi K\varepsilon_0 \frac{L}{\ln(b/a)}$$

where,
L = length [m]
b = radius of the outer conductor [m]
a = radius of the inner conductor [m]

- **Spherical capacitor :**

$$C = 4\pi K\,\varepsilon_0 \left(\frac{ab}{b-a}\right)$$

where,
b = radius of the outer conductor [m]
a = radius of the inner conductor [m]

- **Maximum charge on a capacitor :**

$Q = VC$
where,
Q = charge [C]
V = potential [V]
C = capacitance in farads [F]

- For capacitors connected in series, the charge Q is equal for each capacitor as well as for the total equivalent. If the **dielectric constant** K is changed, the capacitance is multiplied by K, the voltage is divided by K and Q is unchanged. In vacuum $K = 1$ and when dielectrics are used, replace ε_0 with $K\,\varepsilon_0$.

- **Electrical energy stored in a capacitor :** [Joules (J)]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

where, U = Potential Energy [J]
Q = Charge [C]
V = Potential [V]
C = Capacitance [F]

- **Charge per unit area :** [C/m^2]

$$\sigma = \frac{q}{A}$$

where, σ = charge per unit area [C/m^2]
q = charge [C]
A = area [m^2]

- **Energy density :**
- Dimension of energy density is same as pressure.
- Electric energy density is also called Electrostatic pressure.
- Electric force between plates of capacitor

$$F = \frac{1}{2}\varepsilon_0 E^2 \cdot A$$

- Energy stored in terms of Energy density

$$\frac{E}{A \times d} = \frac{1}{2}\varepsilon_0 E^2$$

$$U = \frac{1}{2}\varepsilon_0 E^2$$

Where, U = energy per unit volume [J/m^3]
ε_0 = permittivity of free space, = 8.85×10^{-12} C^2/Nm^2
E = energy [J]

- Capacitors in series :

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$$

- **Capacitors in parallel :**

$$C_{eff} = C_1 + C_2 \ldots$$

- When capacitors are connected in series, all have the same charges.
- When capacitors are connected in parallel, will have the same voltage.
- For parallel capacitors, the total charge is equal to the sum of the charge on each capacitor.
- For series capacitors, the total voltage is equal to the sum of voltages on each capacitor.
- If battery remains connected to a capacitor then potential across capacitor remains same.
- If battery is disconnected then charge stored in capacitor remains same.

❑❑

UNIT - II : Current Electricity

Chapter - 3 : Current Electricity

TOPIC-1

Electric Current, Resistance and Cells

Quick Review

Electric current

- Electric current is defined as the rate of flow of charge, *i.e.*, $I = \frac{dq}{dt}$

- When charge flows at a constant rate, the corresponding electric current can be written as : $I = \frac{q}{t}$
- Conventional current in an external circuit flows from positive terminal to negative terminal.
- Free electrons flow from the negative terminal to the positive terminal in the external circuit.
- 1 ampere = 6.25×10^{18} electrons flow per second.
- Direct current is that which flows in one direction and does not move back and forth while Alternating current is that which flows back and forth and not in a single direction.

Flow of electric charges in metallic conductor

- When an electric charge is applied to a metal at certain points, electrons will move that allows electricity to pass through.
- Without external applied emf, free electrons will move randomly through metal from one point to other giving zero net current.
- Motion of conducting electrons in electric field is a combination of motion due to random collisions.

Drift velocity, mobility and their relation with electric current

- Drift Velocity is an average velocity which is obtained by certain particle like electron due to the presence of electric field.
- Drift velocity is written as :

$$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

where, relaxation time $(\tau) = \frac{\lambda}{v}$, e = charge, m = mass, λ = mean free path

- When electric current is set up in a conductor, electrons drift through the conductor with velocity v_d, is given as

$$v_d = \frac{I}{neA}$$

where,
I = electric current through conductor
n = number density of free electrons
A = area of cross-section
e = charge on electron

- Drift velocity of electrons under ordinary conditions is of the order of 0.1 mm/s.
- Mobility is the drift velocity of an electron for a unit electric field expressed as : $\mu = \frac{V_d}{E}$
- Mobility is always a positive quantity which depends on nature of charge carrier.
- Relation of current and drift velocity is $I = neAv_d$
- Relation between current density and drift velocity is $j = nev_d$

Ohm's law

- Current I in a conductor is directly proportional to the potential difference V applied across the ends of the conductor provided the physical conditions such as the temperature, mechanical strain, etc. remain unchanged.

$$I \propto V$$

$$I = CV \qquad \text{here, C is the conductance of conductor}$$

$$I = \frac{1}{R}V \qquad \left(\text{As } C = \frac{1}{R}\right)$$

or, $V = IR$

where, R = constant known as resistance of conductor

Electrical resistance

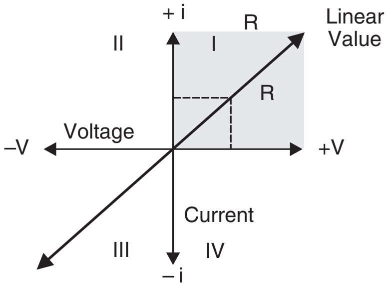
- Electrical resistance is a measure of opposition to the current.
- It is an obstacle that is shown by the conductor during the flow of current as : $R = \frac{m}{ne^2\tau}\frac{l}{A}$
- It is the resistance of the conductor which decreases the current I in time t for a potential difference V applied across its ends.
- The resistance of the conductor is given as : $R = \rho\frac{l}{A}$

where, ρ is specific resistance or resistivity of the material of conductor.

- In the series combination of resistances, the current is same throughout each resistor.
- In the parallel combination of resistances, the potential difference is same across each resistor.

V-I characteristics (linear and non-linear)

- V-I characteristic curves show the relationship between the current flowing through an electronic device and applied voltage across its terminals.



- If electrical supply voltage V applied to terminals of resistive element R varies and resulting current I is measured, then such current is given as $I = \frac{V}{R}$
- V-I characteristic curve shows that on applying voltage to the resistive element, the current will be directly achieved.
- In this, power dissipated by resistive element can also be determined.
- If voltage and current are positive, the V-I characteristic curves will be positive in quadrant I
- If voltage and current are negative, then V-I curve will be shown in quadrant III.

Electrical energy and power

- Electrical energy is that which is stored in the charged particles in an electric field.

$$E = V \times i \times t = i^2 \times R \times t = \frac{V^2}{R} \times t$$

where, E = heat, V = potential difference, t = time, i = current, R = Resistance

- Power is the work done per unit time which is the rate of energy consumed in a circuit.

Since Voltage $V = IR$,

So, $W = (IR) \times It = I^2 R \times t = \left(\frac{V}{R}\right)^2 R \times t = \left(\frac{V^2}{R}\right) \times t$

Hence, $P = \frac{I^2 R \times t}{t} = I^2 R = \left(\frac{V^2}{R}\right)$

where, V = voltage, Q = charge, t = time

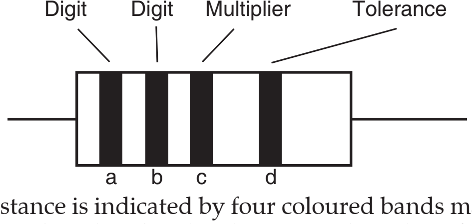
The unit of power is J/s or W.

Electrical resistivity and conductivity

- Resistivity is the specific resistance that is given by the conductor having unit length and unit area of cross-section.
- Conductivity is the reciprocal of resistivity shown as : $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

Carbon resistors and its colour code

- Components and wires are coded with colours to identify their value and function.



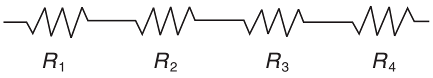
- In carbon resistors value of resistance is indicated by four coloured bands marked on its surface.
- In the carbon resistor shown, first three bands *a*, *b*, *c* determine the value of resistance while fourth band *d* shows tolerance of the resistance.
- Colour bands are marked on body of a carbon resistor to mark its resistance and tolerance in accordance with the following codes :

	Multiplier	Colour	Tolerance
0 Black	10^0	Gold	5%
1 Brown	10^1	Silver	10%
2 Red	10^2	No Colour	20%
3 Orange	10^3		
4 Yellow	10^4		
5 Green	10^5		

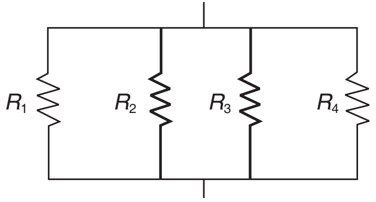
6	Blue	10^6
7	Violet	10^7
8	Grey	10^8
9	White	10^9

Series and parallel combinations of resistors

- If n resistors of resistance R_1, R_2, R_3, ... R_n are connected in series, then their equivalent resistance is given as $R_{eq} = R_1 + R_2 + R_3 + ... + R_n$



- Each resistor in a series circuit has same amount of current flowing through it.



- Voltage drop or power dissipation across each individual resistor in a series is different, and their combined total adds up to the power source input.
- If n resistors of resistance R_1, R_2, R_3, ... R_n are connected in parallel, then their equivalent resistance is given as :
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.

Temperature dependence of resistances

- Resistance depends on arrangement of conductor, material of conductor and on temperature.
- In a resistance model, flow of electrons through a conductor are impeded by atoms and molecules and bouncing of these results difficult for electrons, so resistance increases with temperature.
- With small change in temperature, resistivity varies linearly with temperature as :
$$\rho = \rho_0(1 + \alpha \Delta T)$$
Where, α = temperature coefficient of resistivity.
- By keeping length and area same,
$$R = R_0(1 + \alpha \Delta T)$$
where,
R = resistance at temperature T
R_0 = resistance at temperature ΔT
α = temperature coefficient of resistance for a material
ΔT = change in temperature for which temperature coefficient is specified
- Linear expansion coefficient is much less than temperature coefficient of resistivity.
- As resistor is made from two resistors placed in series where one resistor has positive temperature coefficient while other has negative temperature coefficient.
- **Variation of resistance on stretching a wire :** Consider a wire of length l_1, area of cross-section A_1, volume V, density d and mass m. When it is stretched to length l_2, the area of cross-section changes to A_2. However, the volume, density and mass remain unchanged. Suppose the resistance of the stretched wire be R_2, then

Since, $$R = \rho\frac{l}{A}$$

So, $$\frac{R_2}{R_1} = \frac{A_1}{A_2} \quad \text{(If length } l \text{ is constant)}$$

and $$\frac{R_2}{R_1} = \frac{l_2}{l_1} \quad \text{(If area A is constant)}$$

If the same wire is stretched, then $$A_1 l_1 = A_2 l_2$$

So, $$\frac{R_2}{R_1} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{l_2}{l_1}\right)^2$$

If $A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$, then
$$\frac{R_2}{R_1} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^4 = \left(\frac{l_2}{l_1}\right)^2$$

Internal resistance of cell

- Cell is a device that maintains the potential difference that is present in between the two electrodes as a result of chemical reaction.
- Internal resistance is the resistance that is present in a battery which resists the flow of current when connected to a circuit.
- Internal resistance causes a voltage drop when current flows through it.
- The resistance is provided by the electrolyte and electrodes which are present in a cell that opposes the flow of current in a cell.
- If an internal resistance of a power supply is constant, it is represented as an ideal power supply having fixed output voltage with small resistance connected in series with it.
- If output voltage *V* of a power source is plotted as a function of current *I*, the resulting *V Vs* I plot will give the emf E, of the source, its internal resistance *r*, and maximum current that can be drawn from it.
- As per Ohm's law, voltage dropped across internal resistance is I_r which accounts for the difference between the emf and output voltage of source (E – V), then

$$rI = E - V$$
$$V = -rI + E$$

- The internal resistance of a cell is given as:

$$r = \left(\frac{E - V}{V}\right) \times R$$

It is independent of E and R since V depends on r

Potential difference and emf of a cell

- The emf and terminal potential difference of a cell : Let the emf of a cell be *E* and its internal resistance, *r.* If an external resistance *R* be connected across the cell through a key, then *IR* = *V* = potential difference across the external resistance *R*. This is equal to the terminal potential difference across the cell.

$$E = V + Ir$$
$$I = \frac{E - V}{r}$$

So $$V = E - Ir$$
$$V < E.$$

When current is drawn from a cell, its terminal potential difference is less than the emf.

Combination of cells in series and parallel

- **(i) Series combination of cells :** This combination is used when an external resistance (*R*) of the circuit is much larger as compared to the internal resistance (*r*) of the cell. *i.e.,* $R >> r$.

Let *n* cells, each of emf *E* and internal resistance *r* are connected in series across an external resistance *R,* then the current in the circuit will be

$$I_S = \frac{nE}{R + nr}$$

(ii) Parallel combination of cells : This combination is used when the external resistance *R* is much smaller as compared to the internal resistance (*r*) of the cell, *i.e.,* $R << r$.

When *m* cells are connected in parallel across a resistance *R,* then current through the resistance is given by

$$I_P = \frac{E}{R + r/m} = \frac{mE}{mR + r}$$

If *m* cells of emfs $E_1, E_2, E_3, \ldots E_m$ and of internal resistances $r_1, r_2, r_3, \ldots r_m$ are connected in parallel across an external resistance *R*, then the current through the external resistance is given by

$$I_P = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} \ldots + \frac{E_m}{r_m}}{R + \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots + \frac{1}{r_m}\right)}.$$

(iii) Mixed combination of cells : This type of combination is used when an external resistance *R* is of the same order as the internal resistance *r* of the cell, *i.e.,* $R \approx r$.

Here, $N = nm$ cells are combined in *m* rows, each row having *n* cells, then The current through the external resistance is given by

$$I_m = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr}. \quad ..(i)$$

In eqn. (i) for the value of *I* to be maximum, the value of $(nr + mR)$ should be minimum

Now $$nr + mR = \left(\sqrt{nr} - \sqrt{mR}\right)^2 + 2\sqrt{mnRr}$$

for, $nr + mR$ to be minimum

$$\left(\sqrt{nr} - \sqrt{mR}\right) = 0$$

or $$\sqrt{nR} = \sqrt{mR}$$

or $$nr = mR$$

or $$R = \frac{nr}{m}$$

But, $\frac{nr}{m}$ is the internal resistance of the whole battery.

Thus, in mixed grouping the current in the external circuit will be maximum when the internal resistance of the battery is equal to external resistance. By substituting $\frac{nr}{m} = R$ in eq. (i), we can see that the maximum current in the external circuit will be $\frac{nE}{2R}$ or $\frac{mE}{2r}$

Know the Terms

- **Conductors :** These are materials, which develop electric currents in them, when an electric field is applied.
- **Conventional Current :** The current that flows from a point at higher (positive) potential to a point at lower (negative) potential.
- **Relaxation time :** The short time for which a free electron accelerates before it undergoes a collision with positive ion in the conductor.
- **Drift Velocity :** It is the average speed with which the free electrons in a conductor gets drifted under the influence of external electric field applied across the conductor. It is denoted by $\vec{V_d}$.
- **Electron Mobility :** The mobility of free electrons in a conductor is defined as the drift velocity acquired per unit strength of the electric field applied across the conductor. It is denoted by μ.
- **Conductance :** It is reciprocal of the resistance of a conductor. *i.e.*, $G = \frac{1}{R}$
- **Unit :** ohm^{-1} (Ω^{-1})/siemen(S)/mho.
- **Conductivity :** It is the reciprocal of the resistivity of the material of a conductor *i.e.*, $\sigma = \frac{1}{\rho}$
- **Superconductivity :** The phenomenon, due to which a substance loses all signs of its resistance, when cooled to its critical temperature.
- **Superconductors :** These are the substances which results after superconductivity.
- **Temperature coefficient of resistance :** It is defined as the change in resistance per unit resistance per degree rise in temperature.

Know the Formulae

1.	Electric Current	$i = q/t$
2.	Drift velocity v_d with electric field	$v_d = \frac{-e\vec{E}\tau}{m}$
3.	Current I with drift velocity v_d	$I = n\,e\,\text{A}\,V_d$
4.	Mobility of charge	$\mu = v_d/E = \frac{q\tau}{m}$
5.	Mobility and drift velocity	$v_d = \mu_e E$
6.	Current and Mobility	$\text{I} = neA\,\mu_e E$
7.	Resistance, P. D., and Current	$R = V/I$
8.	Resistance R with specific resistivity	$R = \rho\frac{l}{A}$
9.	Specific resistance, or Resistivity	$\rho = \text{R}\frac{\text{A}}{l}$
10.	Resistivity with electrons	$\rho = m/ne^2\tau$
11.	Current density	$\vec{\text{J}} = \text{I}/\vec{\text{A}}$
12.	Conductance	$G = 1/R$

13.	Conductivity	$\sigma = 1/\rho$
14.	Microscopic form of Ohm's law	$\vec{J} = \sigma\vec{E}$
15.	Temperature coefficient of resistance	$\alpha = \frac{R_t - R_0}{R_0 \times (t_t - t_0)}$
16.	Resistances in series	$R_s = R_1 + R_2 + R_3$
17.	Resistances in parallel	$1/R_p = 1/R_1 + 1/R_2 + 1/R_3$
18.	In a cell, emf and internal resistance	$I = \frac{E}{R+r}$
19.	Potential difference of a cell	$V = E - Ir$
20.	n cells of emf E in series	emf $= nE$
21.	Resistance of n cells in series (Where R is external resistance).	$nr + R$
22.	Current in circuit with n cells in series	$I = \frac{nE}{R+nr}$
23.	n cells in parallel, then emf	emf $=$ E
24.	Resistance of n cells in parallel	$R + \frac{r}{n}$
25.	Cells in mixed group, condition for maximum current	$R = \frac{nr}{m}$
26.	Internal resistance of a cell	$r = \left(\frac{E-V}{V}\right) \times R$
27.	Power of a circuit	$P = VI = I^2R = \frac{V^2}{R}$
28.	Energy consumed	$E = I.V.\Delta T$

TOPIC-2

Kirchhoff's Laws and their Applications

Quick Review

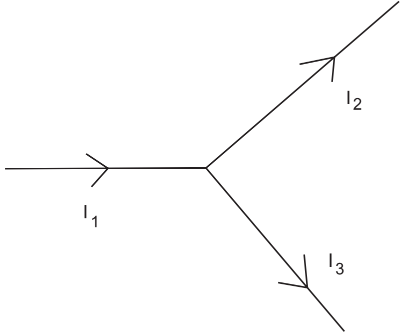
Kirchhoff's Laws

- Kirchhoff's Laws tells about the relationships between voltages and currents in circuits.

First Law

- Kirchhoff's first law is also known as junction rule which states that for a given junction or node in a circuit, sum of the currents entering will be equal to sum of currents leaving.

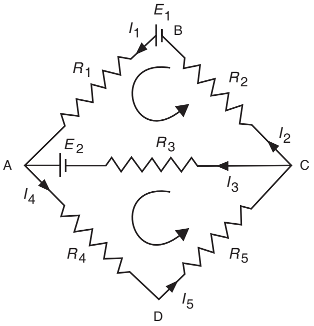
$$I_1 = I_2 + I_3$$



$$I_1 = I_2 + I_3$$

Second Law

- Kirchhoff's second law is also known as loop rule which shows that around any closed loop in a circuit, sum of the potential differences across all elements will be zero.

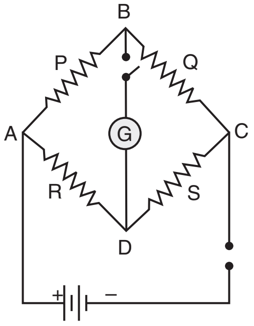


$$\boxed{\Sigma V = 0}$$

Wheatstone Bridge

- Wheatstone Bridge is a circuit having four resistances *P, Q, R* and *S*, a galvanometer and a battery connected as shown.
- It is a balanced bridge where there is no current through the galvanometer and potential at node *B* will be equal to potential at node *D* resulting as :

$$\frac{P}{Q} = \frac{R}{S}$$



Potentiometer

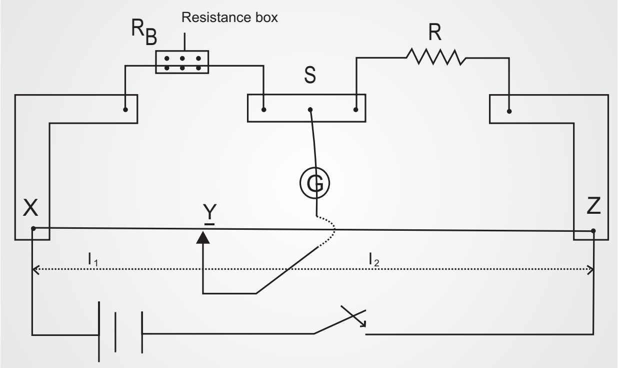
- It is a three terminal resistor where the resistance is manually varied to control the flow of electric current.

- It consists of three terminals among which, two are fixed and one is variable.
- In this, two fixed terminals are connected to both ends of resistive element known as track while the third terminal is connected to sliding wiper.
- In potentiometer, the wiper which moves along the resistive element varies the resistance of the potentiometer.
- The resistance of the potentiometer gets changed when the wiper is moved over the resistive path.

Meter Bridge

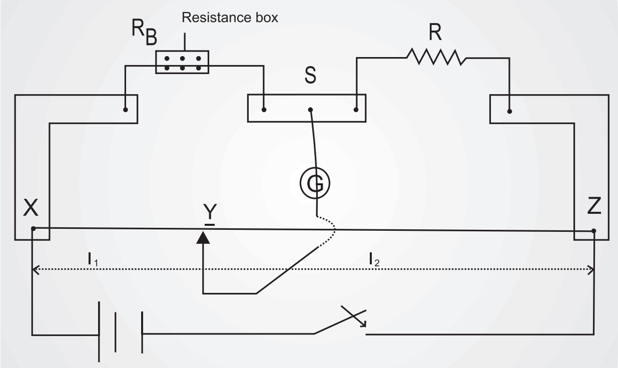
- It is an instrument which is used to find the unknown resistance of a coil or a material connected in a circuit.
- It is also known as slide wire bridge which is an instrument that works on the principle of Wheatstone bridge.
- Meter bridge has two metallic strips which act as holders for the wire that are made of metals like copper.
- **In meter bridge :**
  - Resistance box R_B and unknown resistance R are connected across the two gaps of metallic strips S.
  - One end of galvanometer is connected to the middle lead of metallic strip placed between L shaped strips while other end is connected to a jockey.

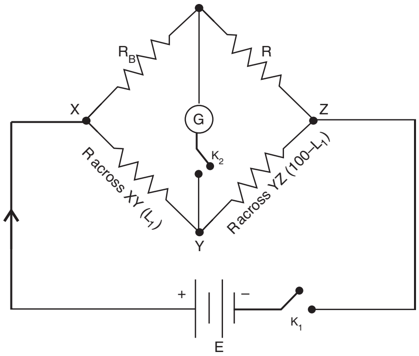
- Jockey which is a metal rod having one end as knife edge is used for sliding on the bridge wire.



From the Meter bridge :

- At negative terminal of galvanometer, there appears zero deflection that makes jockey to connect to negative point on the wire.
- The distance from point *X* to *Y* is taken as L_1 cm while the distance from point *Y* to point *Z* is taken as L_2 cm which can be 100 – L_1 cm.
- Meter bridge can be drawn similar to Wheatstone bridge as :





From the above arrangement :

$$\frac{R_B}{\text{Resistance across } XY} = \frac{R}{\text{Resistance across } YZ}$$

$$\text{Now,} \quad \frac{R_B}{\frac{\rho L_1}{A}} = \frac{R}{\frac{\rho L_2}{A}} \qquad \left[\text{As, } R = \frac{\rho L}{A}\right]$$

$$\text{Further,} \quad \frac{R_B}{\frac{\rho L_1}{A}} = \frac{R}{\frac{\rho(100 - L_1)}{A}} \qquad [\because L_2 = 100 - L_1]$$

$$\text{Hence,} \quad \frac{R_B}{L_1} = \frac{R}{100 - L_1}$$

Know the Terms

- **Conductance :** The reciprocal of resistance with unit as Siemens, "S."
- **Node :** An end point to any branch of a network or a junction common to two or more branches.
- **Permittivity :** The ratio of capacitance between two electrodes with dielectric to capacitance with air between the electrodes.
- **Galvanometer :** An instrument for detecting and measuring small electric currents.

Know the Formulae

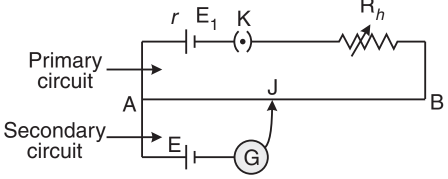
- Kirchhoff's Law (junction rule) $\Sigma i = 0$
  Kirchhoff's Law (Loop rule) $\Sigma V = 0$

TOPIC-3
Potentiometer and its Application

Quick Review

Potentiometer

- Potentiometer is a device which measures the emf of particular cell and helps in comparing the emfs of different cells.
- Potentiometer depends on deflection method where zero deflection results in non drawn of current from the cell or circuit.
- It serves as an ideal instrument of infinite resistance for measuring the potential difference.
- Potentiometer comprises of long resistive wire *AB* of length *L* (about 6 m to 10 m long) made up of manganin or constantan.
- In this, a battery of known voltage *E* and internal resistance *r* forms the primary circuit.
- In the potentiometer circuit, one terminal of other cell is connected at one end of main circuit while other terminal is connected at any point on the resistive wire through galvanometer *G* which forms the secondary circuit.



where,

J = Jockey, *K* = Key, *R* = Resistance of potentiometer wire.

R_h = Variable resistance which controls the current through the wire *AB*

In the circuit :

- Specific resistance (ρ) of wire is high while its temperature coefficient of resistance is low.
- At point *A*, all high potential points of primary and secondary circuits are connected together, while all low potential points are connected to point *B* or jockey.
- Value of known potential difference is more than the value of unknown potential difference that is to be measured.
- The current in primary circuit should remain constant and jockey should not slide with the wire.

Principle of Potentiometer

- Potentiometers are displacement sensors that produce electrical output in proportion to the mechanical displacement.
- It can be used to measure the internal resistance and e.m.f. of a cell which cannot be measured by voltmeter.
- The basic principle of potentiometer is that the potential drop along any length of the wire is directly proportional to its length. So when a constant current flows through a wire of uniform cross-section and composition then,

$$V \propto l.$$

- When there is no potential difference between two points, there will be no flow of electric current.
- Applications of Potentiometer in measuring potential difference and comparing emf of cells Potentiometer in measuring potential difference
- In a potentiometer auxiliary circuit comprises of battery of emf *E* connected across terminals *A* and *B* with rheostat R_h, resistance box and key *K* in series where resistance R_1 is connected to terminal *A* and jockey *J* through galvanometer with cell E_1 and key K_1 in series, then if key K_1 is closed, current will flow through resistance R_1 where a potential difference is developed.
- If J is the position of jockey on potentiometer wire which gets adjusted in such a way that galvanometer shows no deflection, then *AJ* will be the balancing length. *l* on potentiometer wire.
- Here, the Galvanometer will show no deflection as potential difference is same if key *K* is potential gradient of potentiometer wire, then potential difference across resistance R_1 will result as :

$$V = Kl$$

- If r is the resistance of potentiometer of length *L*, then current through potentiometer will be :

$$I = \frac{\varepsilon}{R+r}$$

- Potential drop across potentiometer wire will be :

$$Ir \text{ or } \left(\frac{\varepsilon}{R+r}\right) \times r$$

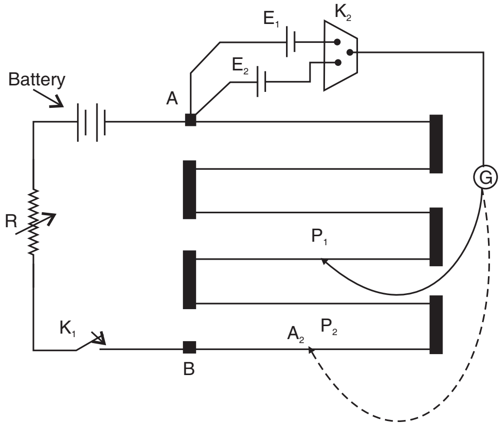
- Now potential gradient of potentiometer wire which is potential per unit length is :

$$K = \left(\frac{\varepsilon}{R+r}\right) \times \frac{r}{L}$$

$$\therefore \qquad V = \left(\frac{\varepsilon}{R+r}\right) \times \frac{rl}{L}$$

Applications of Potentiometric in comparing emf of cells

- If a positive terminal of the cell of emf E_1 is connected to terminal A while negative terminal is connected to jockey by galvanometer, then by closing the key, jockey will move along the wire AB and null point is obtained where galvanometer shows no deflection.



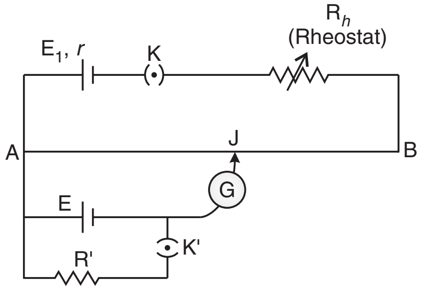
- When the length of wire AP as l_1 is measured, then potential difference across it will balance the emf E_1, So $E_1 = Kl_1$, where K is potential gradient of the wire.
- When cell of emf E_1 is disconnected while cell of emf E_2 is connected, then $E_2 = Kl_2$.
- On comparing and dividing we get an expression :

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

- By knowing the values of l_1 and l_2, the emf of two cells can be compared.

Applications of potentiometer in measurement of internal resistance of cell

(i) Initially in secondary circuit key K' remains open and the balancing length (l_1) is obtained. Since cell E is in open circuit so its emf balances on length l_1, *i.e.* $E = Kl_1$ ...(a)



(ii) Now key K' is closed so cell E comes in closed circuit. If the process is repeated again then the potential difference V balances on length l_2, *i.e.* $V = Kl_2$ ...(b)

(iii) By using formula, internal resistance, $r = \left(\frac{E}{V} - 1\right).R'$

$$r = \left(\frac{l_1 - l_2}{l_2}\right).R' \qquad \text{[Using eqns. (a) and (b)]}$$

Know the Terms

- **Potentiometer :** A potentiometer is an instrument used to measure and compare potentials without drawing any current from the circuit.
- **Current :** Electric current is the rate of flow of charge in a conductor.
- **Potential gradient :** It is the potential difference or fall in potential per unit length of wire.

Know the Formulae

- Potential gradient (K) : $K = \dfrac{V}{L} = \dfrac{iR}{L} = \left(\dfrac{E}{R + R_h + r}\right) \times \dfrac{R}{L}$
- Internal resistance of a cell : $r = \left(\dfrac{E}{V} - 1\right) \times R = \left(\dfrac{l_1 - l_2}{l_2}\right) \times R$
- Comparison of emf's of two cells : $\dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}$

❑❑

UNIT - III : Magnetic Effects of Current and Magnetism

Chapter - 4 : Moving Charges and Magnetism

TOPIC-1

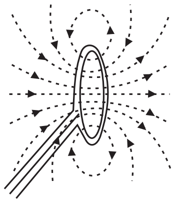
Magnetic Field

Quick Review

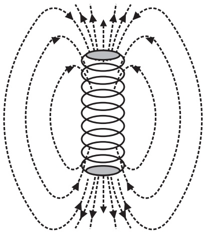
Concept of Magnetic field

- Magnetic field is a region around a magnet where force of magnetism acts which affect other magnets.
- Magnetic field also known as B-field can be pictorially represented by magnetic field lines.
- Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.
- Some of the sources of magnetic fields are :

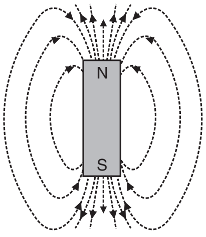
Current in wire



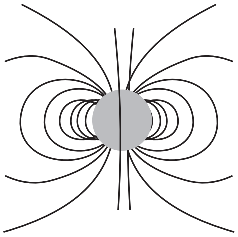
Loop of wire



Solenoid



Bar Magnet



The Earth

- Magnetic field is the amount of force which is exerted on a moving charge where the measurement of force is consistent with Lorentz Force Law as :

$$F = q(v \times B)$$

Where,

F = magnetic force
q = charge
v = velocity
B = magnetic field

- SI unit used to measure magnetic field is Tesla, while smaller magnetic fields are measured in terms of Gauss (1 Tesla = 10,000 Gauss)

$$1 \text{ Tesla} = 10^4\,G$$

$$\frac{\mu_0}{4\pi} = 10^{-7} TA^{-1}\text{m}$$

- When a test charge q_0 enters a magnetic field $\vec{B}$ directed along negative z-axis with a velocity $\vec{v}$ making an angle ϕ with the z-axis, then,

$$\vec{F}_m = q_0(\vec{v} \times \vec{B})$$

$$\text{In } x\text{-direction, } (F_m)_x = q_0 v_y B$$
$$\text{In } y\text{-direction, } (F_m)_y = q_0 v_x B$$
$$\text{In } z\text{-direction, } (F_m)_z = 0$$

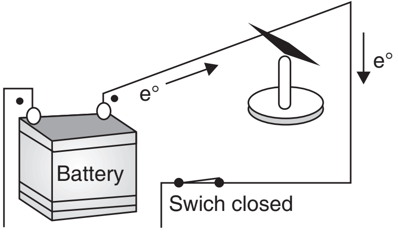
Characteristics of motion of particle in above case will be :

- Speed and kinetic energy of particle do not change, as force is always perpendicular to velocity.
- Direction of velocity will continuously changes, if $\phi = 0$.
- When $\phi = 0$, no force will act on the particle, hence there will be no change in velocity.
- When $\phi = 90°$, test charge describes a circle of radius $\frac{mv}{q_0 B}$, where m is mass of the particle.
- In case of ϕ being any other angle than 0° and 90° test charge will show circular path of radius $\frac{mv\sin\phi}{q_0 B}$ which moves along the direction of magnetic field with speed $v\cos\phi$.
- Momentum along the direction of magnetic field will remain same.
- Angular speed of test charge $\frac{q_0 B}{m}$ is independent of initial speed of particle.
- Centripetal force on test charge $q_0 B\sin\phi$ is independent of the mass of particle.
- Radius of circular path in terms of U_k is given as $R = \frac{\sqrt{2mU_k}}{q_0 B}$; where, U_k is the potential energy.

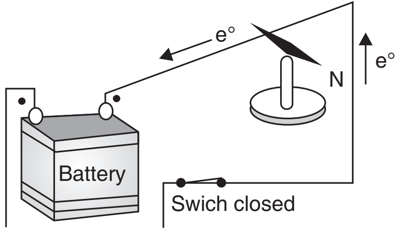
Oersted's experiment

Oersted observed that :

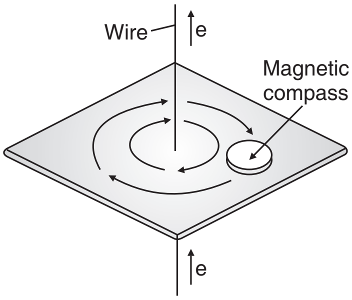
- When there was no current, compass needle below a wire shown in figure will point towards the north.
- When the flow of current is in single direction, then the compass needle below a wire will move at right angles.



- When the flow of current is reversed, compass needle below a wire will move at right angles in opposite direction.



- From an experiment, it is concluded that an electrical current produces a magnetic field which surrounds the wire.



- As per right hand rule, if thumb of right hand is in direction of current, magnetic field will wrap around the wire in same direction with remaining fingers.

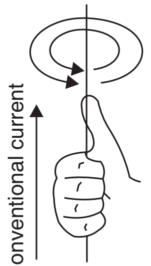


Figure X

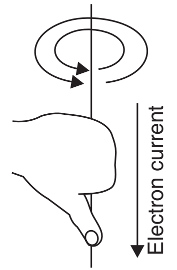


Figure Y

- The right-hand thumb rule is used to determine the direction of magnetic field around conventional current.
- For known flow of electrons, reverse everything and apply left hand thumb rule.

Biot-Savart's law

The magnetic field due to a current element at any nearby point is given by :

$$dB = \left[\frac{\mu_0}{4\pi}\right] I \frac{d\vec{s}\times\vec{r}}{r^3}$$

where,

dB = magnetic field produced by small section of wire

ds = vector length of small section of wire in direction of current

r = positional vector from section of wire to where magnetic field is measured

I = current in the wire

θ = Angle between ds & r

μ_0 = permeability of free space and also $\mu_0 = 4\pi \times 10^{-7}$ Wb/Am^{-}

The magnitude of magnetic field

$$|dB| = \left(\frac{\mu_0}{4\pi}\right)\frac{Idl\sin\theta}{r^2}$$

Applications of Biot-Savart's Law

- Magnetic field at a point in circular loop will be :

$$B = \frac{\mu_0 IR^2}{2\left(R^2+x^2\right)^{\frac{3}{2}}}$$

- Magnetic field at : Centre of the coil

$$B = \frac{\mu_0 Ni}{2R}, \ (x = 0)$$

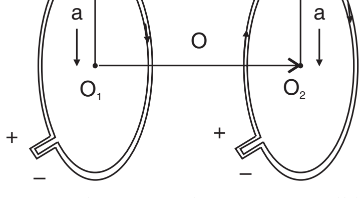
- Magnetic field at very large distance from the centre : $B = \dfrac{2\mu_0 NiA}{4\pi x^3}$ (A = area of circular loop)

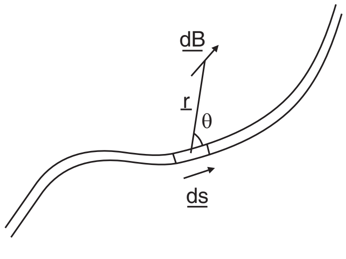
$(R^2 <<< cx^2)$
$(R^2 + x^2 = r^2)$

- Magnetic field at very small distance from the centre : $B = B_{\text{centre}}\left[1 - \dfrac{3x^2}{2r^2}\right]$

- Magnetic field B of Helmholtz coils with similar radius and distance between their centres equals to their radius will be :

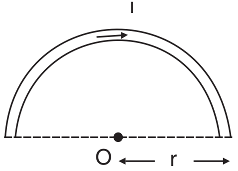
$$B = \frac{\mu_0 NI}{5\sqrt{5}R} = 0.716\frac{\mu_0 NI}{R}$$



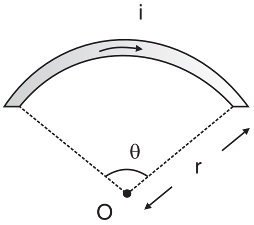


- Magnetic field due to current carrying circular arc with centre O will be :

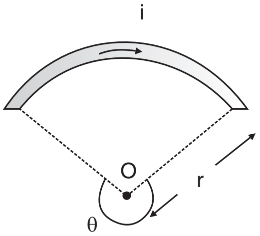
$$B = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{r} = \frac{\mu_0 i}{4r}$$

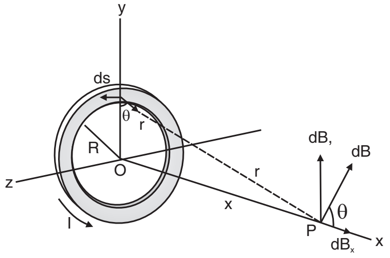


$$B = \frac{\mu_0}{4\pi} \cdot \frac{\theta i}{r}$$



$$B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta)i}{r}$$

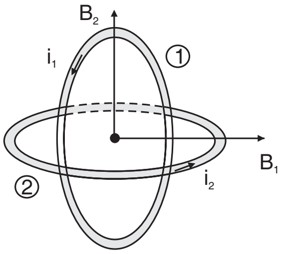




- Magnetic field at common centre of non-coplanar and concentric coils, where both coils are perpendicular to each other will be:

$$B = \sqrt{B_1^2 + B_2^2}$$

$$\frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$$



Know the Terms

- **Magnetic field :** A force field created by moving electric charges and magnetic dipoles that exerts force on charges and dipoles.
- **Static magnetic field :** It is a field created by a magnet that do not vary with time.

Know the Formulae

Biot-Savart's law in Vector Form	Biot-Savart's law in terms of current density	Biot-Savart's law in terms of charge and it's velocity
$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(\vec{dl} \times \vec{r})}{r^3}$ The magnitude of this field $\lvert dB \rvert = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}$ Direction of $\vec{dB}$ is perpendicular to both $\vec{dl}$ and $\hat{r}$. This is given by right screw rule.	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3} dV$ where, $j = \frac{i}{A} = \frac{idl}{Adl} = \frac{idl}{dV}$ = current density at any point of the element, dV = volume of element	$d\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \vec{r})}{r^3}$ $\because \; i\vec{dl} = \frac{q}{dt}\vec{dl} = q\frac{\vec{dl}}{dt} = q\vec{v}$

TOPIC-2

Ampere's Circuital Law and its Applications

Quick Review

- Ampere's Law states that the line integral of H about any closed path is exactly equal to the direct current enclosed by the path, $\int H.dL = I$

- Ampere's circuital law states that the line integral of magnetic field around a closed path is μ_0 times of total current enclosed by the path, $\oint B.dI = \mu_0 I$

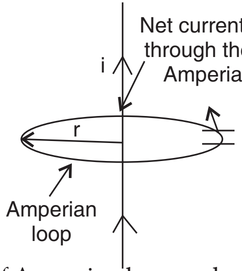
  where,

  B = magnetic field

  dl = infinitesimal segment of the integration path

  μ_0 = empty's permeability

  I = enclosed electric current by the path



- Magnetic field at a point will not depend on the shape of Amperian loop and will remain same at every point on the loop.
- Two parallel wires separated by distance r having currents I_1 and I_2 where magnetic field strength at second wire due to current flowing in first wire is given as :

$$B = \frac{\mu_0 I_1}{2\pi r}$$

  - In this, the field is orientated at right-angles to second wire where force per unit length on the second wire will be :

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

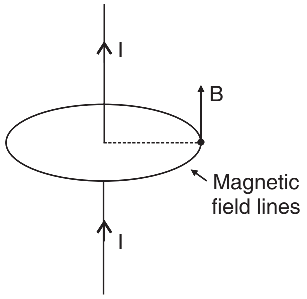
  - Magnetic field-strength at first wire due to the current flowing in second wire will be :

$$B = \frac{\mu_0 I_2}{2\pi r}$$

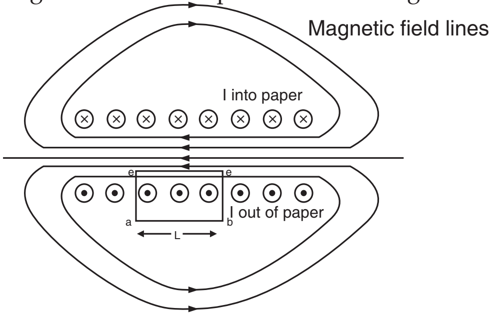
- One ampere is the magnitude of current which, when flowing in each parallel wires one meter apart, results in a force between the wires as 2×10^{-7} N per meter of length

Applications of Ampere's law to infinitely long straight wire; Straight and toroidal solenoids

- Amperes law describes the magnitude of magnetic field of a straight wire as $B = \frac{\mu_0 I}{2\pi r}$



where,
- Field B is tangential to a circle of radius r centered on the wire.
- Magnetic field B and path length L will remain parallel where magnetic field travels.

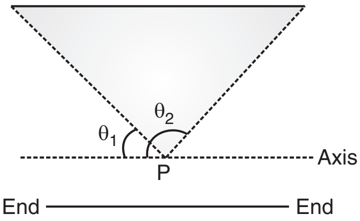


Magnetic Field due to Solenoid

- Solenoid is a tightly wound helical coil of wire whose diameter is small compared to its length.
- Magnetic field generated in the centre, or core of a current carrying solenoid is uniform and is directed along the axis of solenoid.
- Magnetic field due to a Straight Solenoid :
  - at any point in the solenoid, $B = \mu_0 nI$
  - at the ends of solenoid, $B_{end} = \frac{\mu_0 nI}{2}$

  where, n = number of turns per unit length, I = current in the coil

- The current carrying solenoid behaves like a bar magnet whose polarity can be determined by right-hand thumb rule or right-hand first rule similar to current carrying coil.



- Magnetic field at a point on the axis of the solenoid in given as $B = \mu_0 nI[\cos\theta_1 - \cos\theta_2]$
- The magnetic field at the ends of the solenoid ($\theta_1 = 90°$, $\theta_2 = 0°$) is $-\mu_0 nI$ while at any point outside the solenoid, the magnetic field is 0.

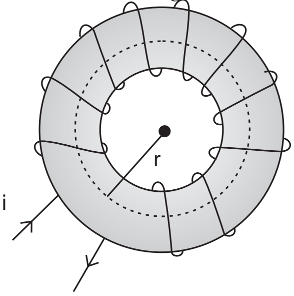
Magnetic Field due to Toroid

- The straight solenoid can be treated as a toroid of infinite radius. Remember, that the magnetic field due to a toroid is independent of its radius.
- A toroid with n turns per unit length with mean radius r, where current i is flowing through it, then the magnetic field experienced by the toroid with total number of turns N will be :

$$\oint \vec{B}.d\vec{I} = \mu_0 i \qquad = B \times 2\pi r = \mu_0 ni$$

Here, $$B = \frac{\mu_0 ni}{2\pi r} = \mu_0 ni \qquad \left(\text{here}, n = \frac{N}{2\pi r}\right)$$

where, r = average radius.



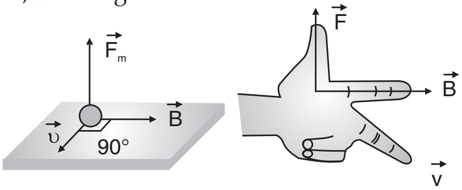
- At any point, empty space surrounded by toroid and outside the toroid, magnetic field B will be zero as net current is zero.

Force on a moving charge in uniform magnetic field :

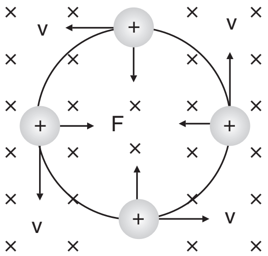
- An electron moving with velocity $\vec{v}$ parallel to flat surface of a magnet experiences a Lorentz force,

$$F = q(\vec{v} \times \vec{B}) = qvB\sin\theta$$

where, v = velocity of the particle, B = magnetic field

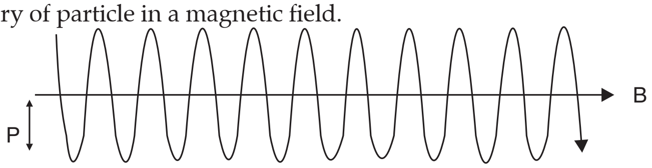


- Force F will always be perpendicular to velocity v and field B in accordance with Right Hand Screw Rule.
- Direction of force on charged particle in magnetic field can also be obtained by Flemings Left Hand Rule.
- Charged particle having charge q and mass m on entering in uniform magnetic field B with initial velocity v perpendicular to field will experience the maximum magnetic force $F_{max} = qvB$ in the direction perpendicular to motion of charged particle.



- When the particles enter the magnetic field as with the same momentum, then radius of path will be $r = \frac{mv}{q_0 B}$

  where, $r \propto \frac{1}{q_0}$
- The circular motion in plane perpendicular to magnetic field and uniform motion along the direction of field will result in spiral trajectory of particle in a magnetic field.

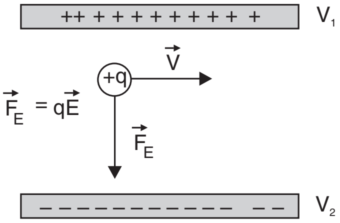


- Charged particle on entering perpendicularly in magnetic field will experience different path directions :

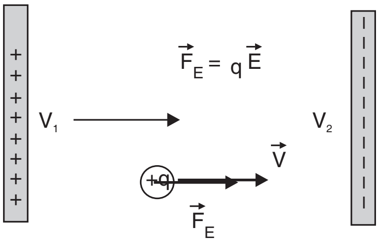
Charge	Magnetic Field Direction	Direction of Motion
Negative	Outward	Anticlockwise
Negative	Inward	Clockwise
Positive	Inward	Anticlockwise
Positive	Outward	Clockwise

Force on a moving charge in uniform electric fields

‹ A charged particle in electric field will experience a force which is independent of its velocity



- If a field is parallel to particle velocity, its direction remains unchanged while velocity increases.



Conservation of total energy will be

$$\Delta U + \Delta K = 0$$

$$\left[q(V_2 - V_1) + \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right] = 0$$

Cyclotron

- The cyclotron consists of two dees in strong magnetic field where oscillating electric field is applied from an oscillator.
- In this, magnetic field is perpendicular to electric field which exists only across the gap between the dees.
- The charged particle gets accelerated, while crossing the gap and moves along the circular paths with radius

$$r = \frac{mv}{q_0 B}$$

- As charged particle is accelerated while crossing the gap, its kinetic energy increases which increases the radius of circular path keeping frequency of revolution unchanged till mass remains constant.
- It cannot be used to accelerate the electrons as electrons move with a velocity very near the velocity of light. Hence, an appreciable increase in mass occurs according to the relation,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- The frequency of cyclotron described in q semicircular path with T as period of oscillating electric field will be :

$$v = \frac{1}{T} = \frac{Bq}{2\pi m}$$

where, $T = 2t = \frac{2\pi m}{qB}$

- Maximum energy gained by the charged particle :

$$E_{\max} = \left(\frac{q^2B^2}{2m}\right)r^2$$

Know the Terms

- **Solenoid :** An electromagnet that generates a controlled magnetic field.
- **Toroid :** It is an electronic component made of hollow circular ring wound with number of turns of copper wire.

Know the Formulae

- Magnetic field at the Surface of a solid cylinder $B = \frac{\mu_0 I}{2\pi R}$
- Magnetic field inside solid cylinder : $B = \frac{\mu_0 Ir}{2\pi R^2}$
- Magnetic field due to infinte sheet carrying current : $B = \frac{\mu_0 j}{2}$
- Magnetic field inside the solenoid at a point : $B = \frac{\mu_0}{4\pi}(2\pi In)[\sin\alpha + \sin\beta]$

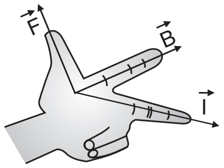
TOPIC-3

Current Carrying Conductor and Galvanometer

Quick Review

Force on a current-carrying conductor in a uniform magnetic field

- The magnetic force on current-carrying conductors is used to convert electric energy to work.
- The magnetic force on current-carrying conductors is given by $F = IBl \sin\theta$
  where, I= current, l= length of straight conductor in uniform magnetic field B, θ = angle between I and B.
- The direction of the force is always right angles to the plane containing the conductor and magnetic field given by Fleming's Left-Hand Rule.

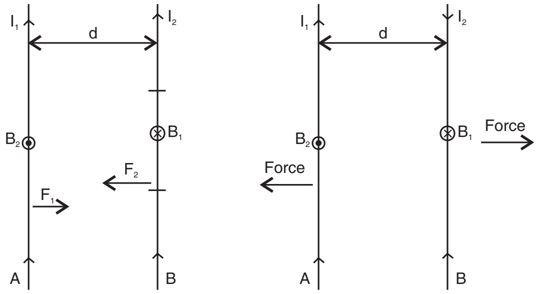


- Factors that affect the magnetic force in a magnetic field includes, strength of magnetic field, current flowing through wire and length of wire.
- When conductor is perpendicular to magnetic field, force will be maximum and when it is parallel, force will be zero.

Force between two parallel current-carrying conductors

- Two current carrying conductors will attract each other when current flows in similar direction and will repel each other when current flows in opposite direction
- In two long parallel wires separated by distance d with currents I_1 and I_2, if flow of currents is in same direction then, wire B will experience a magnetic field due to wire A given as $B_1 = \frac{\mu_0 II_1}{2\pi d}$ with force per unit length $\frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$.
- Wire A will experience a force due to magnetic field B_2 of wire B and this force F_1 will have a magnitude equal to that of F_2 in opposite direction making wire A to attract towards wire B.
- Force per unit length of wire B is

$$\frac{F_2}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$



Similarly force per unit length of A due to current in B is

$$\frac{F_1}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

and is directed opposite to the force on B due to A. Thus the force on either conductor is proportional to the product of the current.

Definition of Ampere

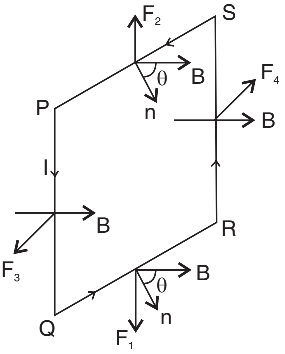
- Ampere is the force between two current carrying wires in uniform magnetic field.\
- When two long parallel wires 1 m apart each carry a current of 1 A, then force per unit length on each wire will be 2×10^{-7} N/m.

Torque experienced by a current loop in uniform magnetic field

- In a rectangular loop of length l, breadth b with current I flowing through it in a uniform magnetic field of induction B where angle θ is between the normal and in direction of magnetic field, then the torque experienced will be :

$$\tau = nBIA\sin\theta$$

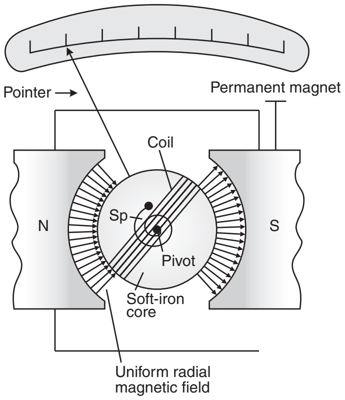
where, n = number of turns in the coil
Further, $\tau = mB\sin\theta \quad \therefore nIA = m$



- Torque will be maximum when the coil is parallel to magnetic field and will be zero when coil is perpendicular to magnetic field.
- In vector notation, torque $\vec{\tau}$ experienced will be $\vec{\tau} = \vec{m} \times \vec{B}$

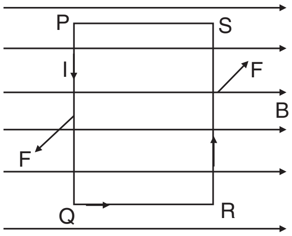
Moving coil galvanometer

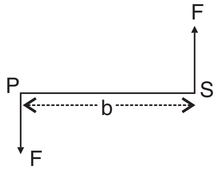
- It is an instrument used for detection and measurement of small electric currents.
- In this, when a current carrying coil is suspended in uniform magnetic field, it experience a torque which rotate the coil.
- The force experienced by each side of the galvanometer will be $F = BIl$ which are opposite in direction.



- Opposite and equal forces form the couple which generates deflecting torque on the coil having number of turns *n* is given as :

$$\tau = F \times b = nBIl \times b = nBIA$$





- In moving coil galvanometer, current in the coil will be directly proportional to the angle of the deflection of the coil, $I \propto \phi$. where, ϕ is the angle of deflection.

Current sensitivity of galvanometer

- Current sensitivity of galvanometer is the deflection produced when unit current passes through the galvanometer. A galvanometer is said to be sensitive if it produces large deflection for a small current.

$$I = \frac{C}{nBA}\theta$$

$$\frac{\theta}{I} = \frac{nBA}{C}$$

- Voltage sensitivity of galvanometer is the deflection per unit voltage given as

$$\frac{\theta}{V} = \frac{\theta}{IG} = \frac{nBA}{CG}$$

where, G = galvanometer resistance, C = torsional constant.

(i) Increase in sensitivity of moving coil galvanometer depends on :

(ii) number of turns N, magnetic field B, area of coil A and **(iii)** torsional constant.

Conversion of galvanometer into ammeter

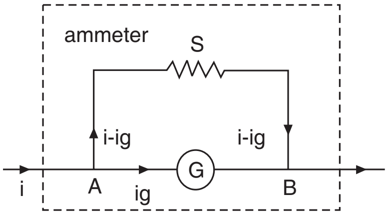
- Galvanometer can be converted into ammeter by connecting a low resistance known as shunt in parallel with the galvanometer coil.
- If I_g being the maximum current with full scale deflection that passes through galvanometer, then current through shunt resistance will be $i_s = (i - i_g)$

where, G = Galvanometer resistance, S = Shunt resistance and i = Current in circuit

- Now effective resistance of ammeter will be :

$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S}$$

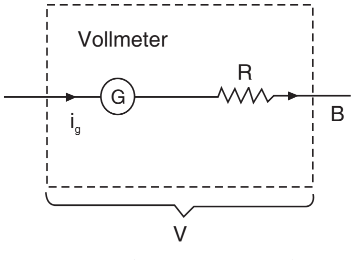
$$R_a = \frac{GS}{G+S}$$



Conversion of galvanometer into voltmeter

- Voltmeter measures the potential difference between the two ends of a current carrying conductor.
- Galvanometer can be converted to voltmeter by connecting high resistance in series with galvanometer coil.
- As resistance R is connected in series with galvanometer, current through the galvanometer will be,

$$i_g = \frac{V}{R+G} \quad or, \quad R = \frac{V}{i_g} - G$$



- Effective resistance of voltmeter is $R_v = G + R$, where R_v is very large making the voltmeter to connect in parallel since it can draw less current from the circuit.

Know the Terms

- **Moving coil galvanometer :** An instrument used for detection and measurement of small electric currents.
- **Current sensitivity of galvanometer :** A deflection produced for a unit current flowing through it.
- **Ammeter :** Device used for measuring the current flowing through a conductor.

Know the Formulae

- $F = \frac{\mu_0}{4\pi} \times \frac{2i_1i_2}{a} \times l$
- $F_m = \frac{\mu_0}{4\pi} \times \frac{q_1q_2v_1v_2}{r^2}$
- $\tau_{\max} = NBiA$
- $S_v = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC}$

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Chapter - 5 : Magnestism and Matter

TOPIC-1

Magnetic Dipole

Quick Review

Current loop as a magnetic dipole and its magnetic dipole moment

- Magnetic dipole is a small magnet of microscopic dimensions similar to flow of electric charge around a loop.
- Magnetic dipole moment is the strength of magnetic dipole that measures dipole's ability to align itself with external magnetic field.
- Magnetic dipole moment known as magnetic moment is the maximum amount of torque caused by magnetic force on dipole which appears per unit value of surrounding magnetic field in vacuum.
- Magnetic field produced at large distance *r* from the centre of circular loop along its axis will be

$$B = \frac{2\mu_0 IA}{4\pi r^3}$$

where, I = current in the loop
A = area

- Magnetic moment of current loop is the product of current and loop area,

$$M = I \times A$$

- A current loop may experience a torque in a constant magnetic field,

$$\vec{\tau} = \vec{M} \times \vec{B}$$

Magnetic dipole moment of revolving electron

- For an electron of charge e revolving around a nucleus of charge Z_e at an orbit of radius r, with velocity v and magnetic moment μ, the orbital magnetic moment will be

$$\mu_1 = -\frac{em_evr}{2m_e}$$

But angular momentum of electron

$$L = m_evr$$

$$\therefore \qquad \mu_1 = -\frac{e}{2m_e}L$$

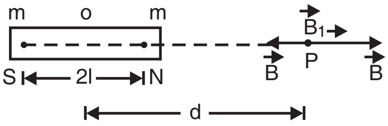
Here (–) sign shows that angular momentum's direction is opposite to the magnetic moment's direction.

Magnetic field intensity due to magnetic dipole (bar magnet) along its axis and perpendicular to its axis

- Bar magnet has two magnetic poles known as north pole and south pole each having strength m. If the separation between the poles known as magnetic length is l, then bar magnet is said to have magnetic dipole moment which is $$M = ml$$
- Direction of magnetic dipole moment is from South Pole of bar magnet to North Pole of bar magnet.
- Horizontal component of earth's magnetic field at the poles is zero.
- Lines of magnetic force runs in closed loops and continuously, both inside and outside a bar magnet.

Point lies on axis line of bar magnet :

- If $2L$ is length of bar magnet, m is magnetic strength of each pole, M is dipole, then magnetic field at point P due to :



North pole of magnet (N)

$$B = \frac{\mu_0 m}{4\pi(d-l)^2}$$

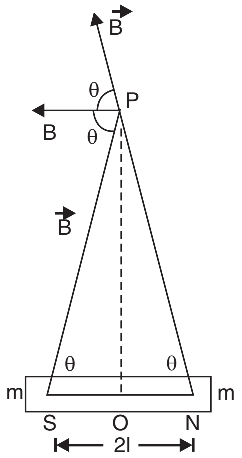
South pole of magnet (S)

$$B = \frac{\mu_0 m}{4\pi(d+l)^2}$$

Hence, resultant at point P when $2l << d$:

$$B = \frac{\mu_0 2M}{4\pi d^3}$$

Point lies on equatorial line of bar magnet :



- If $2L$ is magnet length of bar magnet, m is magnetic strength of each pole, M is dipole, then magnetic field at point P due to :

  N pole of magnet (direction N-P)

$$B = \frac{\mu_0 m}{4\pi(d^2+l^2)}$$

- N pole of magnet (direction P-S)

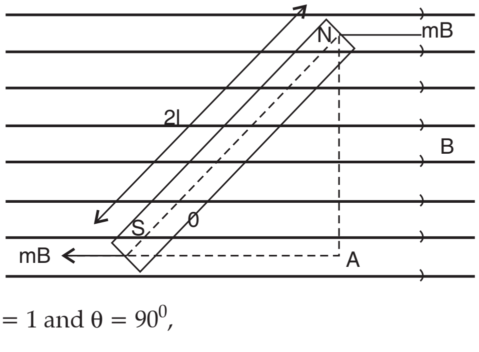
$$B = \frac{\mu_0 m}{4\pi(d^2+l^2)}$$

- Hence resultant at point P when $2l << d$:

$$B = \frac{\mu_0 M}{4\pi d^3}$$

Torque on a magnetic dipole (bar magnet) in uniform magnetic field

- A bar magnet with length $2l$ and pole strength m in uniform magnetic field induction B at angle q with force mB acting on North and South Pole along the direction opposite to magnetic field results as a couple where torque τ due to couple will be :



τ = Force × perpendicular distance

$\tau = F \times NA$

$= mB \times NA$

$= mB \times 2l\sin\theta$

$= MB\sin\theta$

In vector form :

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of τ is perpendicular to plane containing, so when $B = 1$ and $\theta = 90^0$,

$$\tau = M$$

- Magnetic moment M of the magnet will be equal to the torque required to keep the magnet at right angles to a magnetic field of unit magnetic induction.

Know the Terms

- **Magnetic dipole :** A system of two equal and opposite poles separated by small distance.
- **Magnetic length :** It is the distance between the two poles of a magnetic dipole.
- **Magnetic dipole moment :** It is the product of pole strength and separation between two poles and is denoted by M.
- **Geographic meridian :** It is a vertical plane that passes through the north and south poles at a given place.
- **Magnetic declination :** It is a region which describes the angle between the geographic meridian and magnetic meridian.
- **Magnetic intensity** : It is the magnetic moment per unit volume.

Know the Formulae

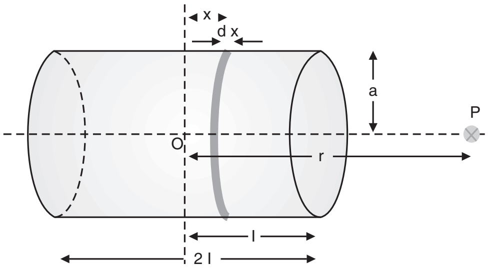
- Magnetic field due to short dipole at distance *'a'* on axial line: $B_{axisl} = \frac{\mu_0}{4\pi}\frac{2M}{a^3}$
- Magnetic field due to short dipole at distance *'a'* on equatorial line: $B_{equi} = \frac{\mu_0}{4\pi}\frac{M}{a^3}$
- Torque on a magnetic dipole in uniform magnetic field : $\tau = MB \sin\theta$

TOPIC-2
Magnets

Quick Review

Bar magnet as an equivalent solenoid

- If a solenoid of length $2l$, radius a with current I having n number of turns per unit length, then the magnetic moment of solenoid m (= NIA), $B = \frac{\mu_0 2M}{4\pi r^3}$



- Magnetic moment of a bar magnet is equal to magnetic moment of an equivalent solenoid that produces same magnetic field.

Magnetic field lines

- Magnets have two poles where field lines spread from North Pole and circle back around to South Pole.
- The region of space around a magnet, current carrying conductor or moving charge, in which magnetic effect can be experienced, is called a magnetic field.
- Magnetic field lines are imaginary lines used to visualize magnetic field.
- The tangent at any point on a field line gives the direction of magnetic field at that point
- They are crowded near poles of the magnet and far from the magnet, where the magnetic field is weak.

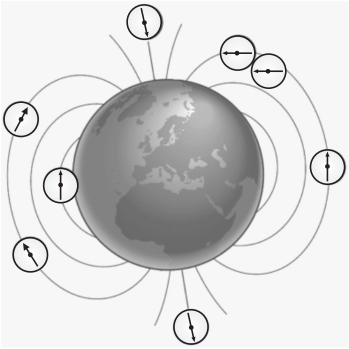
Gauss's Law for Magnetic Fields

- Gauss' Law for magnetism applies to the magnetic flux through a closed surface.
- Shows that no magnetic monopoles exist and total flux through closed surface will be zero.
- The Gauss's law for magnetic fields in integral form is given by $\phi = \int B.dA = 0$

Earth's magnetic field

- Earth's magnetic field is also known as geomagnetic field which extends from Earth's interior into space meeting with solar wind.
- Earth's magnetic field is same as that of bar magnet which is tilted 11° from spin axis of Earth.

- Earth's magnetic field is attributed to dynamo effect of circulating electric current having variable direction.
- Similar to bar magnet, earth's magnetic field gets changed with time as it is generated by geodynamo.



Magnetic elements; Para-, dia- and ferro-magnetic substances

Ferromagnetism

- The substances, which are placed in external magnetising field, get magnetised very strongly in the direction of the magnetising field are called ferromagnetic. The examples of ferromagnetic substances are : iron, nickel, cobalt and certain alloys such as alnico.
- Ferromagnetism is not found in liquids and gases.

  The ferromagnetism depends on the temperature. If we raise temperature of the ferromagnetic substance, the domains may disintegrate and the substance turns paramagnetic. The temperature at which this happens is called Curie point. Thus, the Curie point is the temperature above which the ferromagnetic substance looses ferromagnetism and turns into a paramagnetic substance.
- Ferromagnetic substances are strongly magnetised and their induced dipole moment p_m is in the direction of the magnetising field H.
- For ferromagnetic substances. $\mu_r > > 1$
- χ_m of ferromagnetic substance is positive and large.
- B (magnetic induction) inside the ferromagnetic substances is much larger than that in vacuum. *B* for ferromagnetic substances is much larger than that for paramagnetic substances.
- They move from the weaker regions of the magnetic field to the stronger regions.
- They are strongly attracted by a bar magnet.
- If a bar magnet of ferromagnetic materials is freely suspended between the pole pieces of a magnet, it aligns itself along the magnetic field.

  The liquids do not exhibit ferromagnetism. However, if a finely powdered ferromagnetic substance is placed in the watch glass kept on the pole pieces of a magnet, it elevates from the middle when pole pieces are closed together which suffers a depression in the middle when the pole pieces are far apart.

  Area under a B-H curve represents power loss per unit volume per unit unit cycle.

  Diamagnetism :
- The substances, which when placed in external magnetising field, get magnetised feebly in a direction opposite to the magnetising field are called diamagnetic substances.

  Examples of diamagnetic substances : Copper, silver, gold, bismuth, zinc, lead, glass, marble, sodium chloride, water and gases such as helium, argon, etc.
- Diamagnetic substances are feebly magnetised and their induced magnetic dipole moment is directed opposite to the magnetising field H.
- For diamagnetic substances, $0 \leq \mu_r < 1$.
- χ_m of diamagnetic substances is negative and small, $-1 \leq x_m < 0$.
- B (magnetic induction field) inside the diamagnetic materials is less than that in vacuum, because,

$$B = \mu_0 (H + M)$$
$$B = \mu_0(H + \chi_m H) = \mu_0(1 + \chi_m)H.$$

Because χ_m is negative, therefore *B* is less than *H*

$$B_0 = \mu_0 H$$

- They move from the stronger regions of magnetising field to the weaker regions.
- They are repelled by a bar magnet.
- If a bar of diamagnetic material is suspended between the pole pieces of a magnet, it aligns itself perpendicular to the magnetic field of the magnet.
- Diamagnetic substances do not obey the Curie's law.
- Diamagnetic properties are independent of the temperature.

Paramagnetism :

- The substances which when placed in an external magnetising field, get weakly magnetised in the direction of magnetic field are called paramagnetic substances.

  Examples of paramagnetic substances : Aluminium, antimony, platinum, manganese, sodium, chromium, copper chloride, liquid oxygen, etc.
- Paramagnetic substances are feebly magnetised and their induced magnetic dipole moment p_m is in the direction of the magnetising field H.
- For paramagnetic substance $\mu_r > 1$ but small, $1 < \mu_r < 1 + \varepsilon$.
- χ_m of paramagnetic substances is positive but small, $0 < x_m < \varepsilon$.
- B (magnetic induction field) inside the paramagnetic materials is slightly more than that in vacuum.
- They move from the weaker regions of magnetic field to the stronger regions.
- They are attracted by a bar magnet.
- If a bar magnet of paramagnetic material is suspended between the pole pieces of the magnet it aligns itself parallel to the magnetising field of the magnet.
- The paramagnetism depends on the temperature. As the temperature increases, magnetisation tends to decrease.
- Paramagnetic substances obey the Curie's law, which states that the magnetic susceptibility is inversely proportional to the temperature in kelvin, *i.e.*, $\chi_m \propto \frac{\text{I}}{\text{T}}$.
- Angle between the geographic meridian and the magnetic meridian is called declination.
- Angle between the horizontal component and the total magnetic field of the earth is called dip.
- Dip is zero at the equator and 90^{o} at the magnetic poles.
- Paramagnetism and ferromagnetism are associated with the intrinsic magnetic moment of spinning of electrons.
- In ferromagnetism, the magnetic effect is enhanced due to the formation of domains.
- Magnetic induction B and magnetic intensity H are related as $B = \mu H$.
- Magnetic induction is expressed in tesla (S.I.) and gauss (C.G.S. system).
- Magnetic intensity in vacuum is expressed in Oersted (C.G.S. system) and Am^{-1} in *SI* unit.
- Diamagnetism originates from the magnetic moment associated with the orbital motion of electrons.
- **Curie's Law in Magnetism :** The intensity of magnetisation I of a paramagnetic material is directly proportional to the strength of the external magnetic field H, called the magnetising field and is inversely proportional to absolute temperature of the material.

$$\text{I} \propto \frac{H}{T}$$

$$\therefore \qquad \text{I} = \frac{CH}{T}$$

$$\Rightarrow \qquad \frac{I}{H} = \frac{C}{T}$$

$$\text{or} \qquad \chi = \frac{C}{T}$$

where, C is known as Curie constant.

Magnetic Permeability : Magnetic field exists in vacuum. It can exist in other materials also. Magnetic permeability of a medium is defined as the ratio of magnetic intensity and the magnetising field, *i.e.*,

$$\mu = \frac{B}{H}$$

Magnetic permeability of a material decides the ability of the material to allow magnetic lines of force through it.

Electromagnets and factors affecting their strengths

- Electromagnet is also known as a solenoid having a core of iron with wire wrapped around on it.
- Factors which affect the strength of electromagnets are :
  - nature of core material
  - strength of current passing through the core
  - number of turns of wire on the core
  - shape and size of the core

- Core is a solenoid having copper wire wound on it. More will be the number of turns, more will be the strength of magnetic field.

Permanent magnets

- Magnets where magnetic field is generated by internal structure of material itself.
- Permanent magnets have consistent non-varying magnetic field having north and south pole linked with it.
- Magnetic fields of permanent magnets are the sum of nuclear spins, electron spins and orbits of electrons.
- Permanent magnets produce a longitudinal magnetic field between the poles.

Advantages	Disadvantages
• No power supply needed	• Direct field only
• Cling to vertical surfaces	• Deteriorate with wear
• No electrical contact problems	• Have to be pulled from test surface
• Inexpensive	• No control over field strength
• No damage to test piece	• Magnetic particles attracted to poles
• Lightweight	• Legs must have area contact
	• May have to be recharged

Know the Terms

- **Magnetic lines of force :** The lines that are mapped out around the magnet.
- **Permanent magnet :** A material, or piece of such material, which retains its magnetism even when not subjected to any external magnetic fields.
- **Ferromagnetic :** A material, such as iron or nickel, that is easily magnetized.
- **Electromagnet :** A magnet which attracts metals only when electrically activated.
- **Hysteresis :** The intensity magnetisation (M) or magnetic induction (B) of the ferromagnetic substances lags behind the magnetising field H. The phenomenon is called hysteresis.

Know the Formulae

- $$\tau = M \times B$$
- $$\tau = MB \sin\theta$$

  where, θ is the angle between $\vec{M}$ and $\vec{B}$
- $$\chi_m = M/H$$
- $$\chi_m = \mu_r - 1$$
- $$I \propto \frac{H}{T}$$
- $$\mu = \frac{B}{H}$$
- $$\begin{aligned} B &= \mu_0 (H + M) \\ &= \mu_0 (H + \chi_m H) = \mu_0 (1 + \chi_m) H \end{aligned}$$
- The force between two magnetic poles of strength $q_m = m$ and $q_{m'} = m'$, separated by a distance r is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{q_m q_m{}'}{r^2}$$

Factors affecting the strength of electromagnets

1. Nature of core material.
2. Strength of Current Passing through the core.
3. Number of turns of wire on the core.
4. Shape and size of the core.

Characteristic	Diamagnetic	Paramagnetic	Ferromagnetic
μ_r	< 1	> 1	$>> 1$
χ_m	< 0	> 0	$>> 0$
		small	large

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UNIT - IV : Electromagnetic Induction and Alternating Currents

Chapter - 6 : Electro-Magnetic Induction

TOPIC-1

Magnetic Flux and Induction

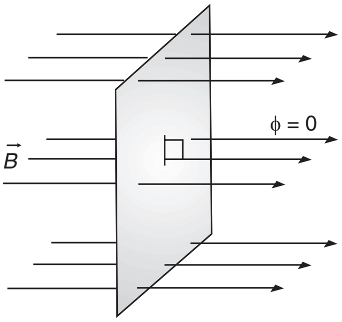
Quick Review

Electromagnetic induction

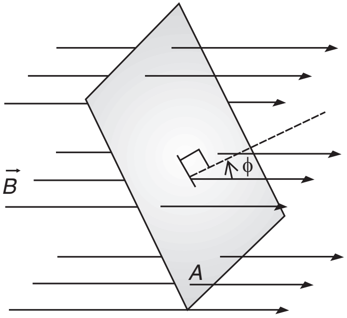
- Electromagnetic induction is the process of generating electric current with a magnetic field.
- It takes place whenever a magnetic field is changing and electric conductor moves relative to one another where conductor crosses lines of force in magnetic field.
- The current produced by electromagnetic induction is more when the magnet or coil moves faster and when magnet or coil moves back and forth repeatedly, then alternating current is produced.

Magnetic Flux :

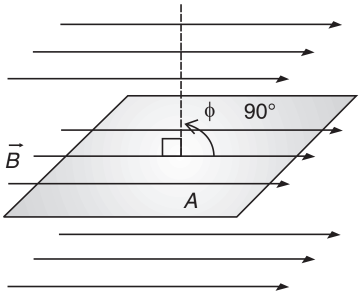
- Magnetic flux through an enclosed area is the amount of field lines cutting through a surface area *A* defined by unit area vector.
- The units for magnetic flux are webers, where, 1Wb=1T·m^2.
- Magnetic flux ϕ_B is related to number of field lines passing through a given area.
- If magnetic field is changing, then changing magnetic flux will be $\phi_B = NBA \cos\theta$, where angle θ is the angle between field and normal to the plane.
- The induced EMF is related to rate of change of magnetic flux known as Faraday's Law



$\vec{B}$ perpendicular to A (ϕ=0) :
magnetic flux Φ_B= BA.



$\vec{B}$ at an angle ϕ with the perpendicular to A:
magnetic flux Φ_B= BA cos



$\vec{B}$ parallel to A ($\phi = 90$):
magnetic flux Φ_B= 0.

Magnetic flux density

- The change in magnetic flux per unit change in area is called magnetic flux density.
- Magnetic flux is given by : $df = \vec{B}\cdot\vec{dA}$. For $\vec{B}$ parallel to $\vec{dA}$, we have

$$df = B(dA)\cos 0° = B(dA)$$

Therefore,
$$B = \frac{d\phi}{dA} \qquad \text{(i)}$$

i.e., magnetic induction is equal to the magnetic flux density. In other words, the magnetic field may be measured in terms of magnetic flux density. From equation (i), we find :

Unit of
$$B = \frac{\text{Unit of } d\phi}{\text{Unit of } dA}$$

Or
$$T = \frac{\text{Wb}}{\text{m}^2}$$

i.e., Tesla = weber per square metre.

Faraday's Laws of Electromagnetic Induction

- The induced emf in a closed loop due to a change in magnetic flux through the loop is known as Faraday's law.
- **Faraday's First Law** of Electromagnetic Induction states that whenever a conductor is placed in varying magnetic field emf is induced which is known as induced emf and if the conductor circuit is closed current are also induced which are called induced current.
- **Faraday's Second Law** of Electromagnetic Induction states that the induced emf is equal to the rate of change of flux linkage where flux linkage is nothing but the product of number of turns in the coil and flux associated with the coil.

$$\varepsilon = \frac{d\phi_s}{dt}$$

ϕ_B is magnetic flux through the circuit as $\phi_B = \int \vec{B}.d\vec{A}$

With N loops of similar area in a circuit and ϕ_B being the flux through a loop, then emf is induced in every loop making Faraday law as

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

where, ε = Induced emf [V], N = number of turns in the coil
$\Delta\phi$= change in the magnetic flux [Wb], Δt = change in time [s]
The negative sign means that ε opposes its cause.

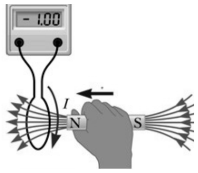
- If there is no change in magnetic flux, no induced emf is induced.

Induced emf and current

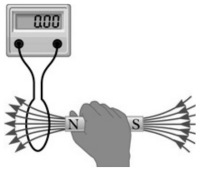
- A changing magnetic flux induces an electric field which induces a current in the circuit
- A wire moving in the field induces a current which acts same as current created by a battery
- Changing magnetic flux and induced electric field are related to induced emf as per Faraday's law.
- The induced EMF is related to the magnetic field as $E = B.l.v \sin\theta$

Induced Current

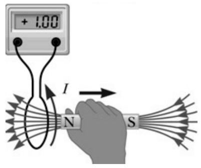
- When a conductor moves across flux lines, magnetic forces on the free electrons induce an electric current.
- When a magnet is moved towards a loop of wire connected to ammeter, in such case, ammeter shows current induced in the loop.



- When a magnet is held stationary, there will be no induced current in the loop, if also the magnet is inside the loop.



- When a magnet is moved away from the loop, in such case, ammeter shows opposite current induced in the loop.

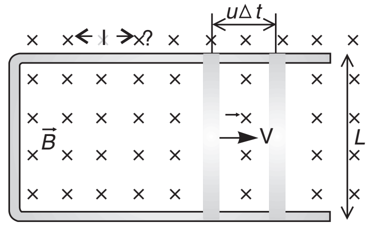


Motional Emf

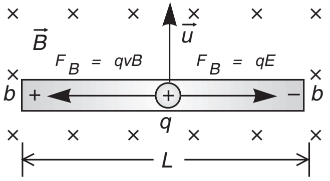
- The relationship between an induced emf ε in a wire moving at a constant speed v through a magnetic field B is given by :

$$\Delta\phi_B = BLv\Delta t$$

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = Blv$$

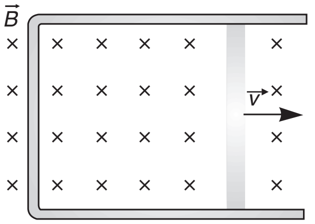


- An induced emf from Faraday's law is created from a motional emf that opposes the change in flux.
- Magnetic and electric forces on charges in a rod moving perpendicular to magnetic field is given as :

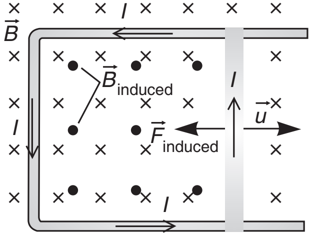


Lenz's Law

- Lenz's law is used to determine the directions of induced magnetic fields, currents, and emfs.
- The direction of an induced emf always opposes the change in magnetic flux that causes the emf.
- It explains about the negative sign in Faraday's flux rule, $\varepsilon = -\frac{d\phi_B}{dt}$ showing that the polarity of induced emf tends to produce a current that opposes the change in magnetic flux that produced it.
- As per conservation of energy, induced emf opposes the cause which produces it making mechanical work to continue with the process which gets converted into electrical energy.
- Slide wire moving in magnetic field :



- Slide wire containing induced current, magnetic field and magnetic force :



Electric Generators and Back Emf

- Electric generator rotates a coil in a magnetic field inducing an emf which is given as a function of time $\varepsilon = NBA\omega\sin(\omega t)$
  where,
  A = area of N-turn coil rotated at constant angular velocity ω in uniform magnetic field $\vec{B}$.
- The peak emf of a generator is $\varepsilon_0 = NBA\omega$
- Any rotating coil produces an induced emf. In motors, it is known as back emf as it opposes the emf input to the motor.

Know the Terms

- **Electric generator :** Device for converting mechanical work into electric energy that induces an emf by rotating a coil in magnetic field.
- **Induced electric field :** Field created due to changing magnetic flux with time.
- **Induced emf :** It is a short-lived voltage generated by a conductor or coil moving in a magnetic field.
- **Lenz's law :** Lenz' law states that the direction of an induced emf is such that it opposes the change in magnetic flux which produces it.
- **Magnetic damping :** It is a drag which is produced by eddy currents.

- **Magnetic flux :** It is the amount of magnetic field lines that is measured through a given area.
- **Motionally induced emf :** Voltage produced by movement of conducting wire in a magnetic field.
- **Peak emf :** It is the maximum emf produced by a generator.

Know the Formulae

- Magnetic flux : $\phi_m = \int_s \vec{B}.\hat{n}dA$
- Faraday's law : $\varepsilon = -N\dfrac{d\phi_m}{dt}$
- Motionally induced emf : $\varepsilon = Blv$
- Motional emf around a circuit : $\varepsilon = \oint E.dl = -\dfrac{d\phi_m}{dt}$
- emf produced by an electric generator : $\varepsilon = NBA\omega\sin(\omega t)$

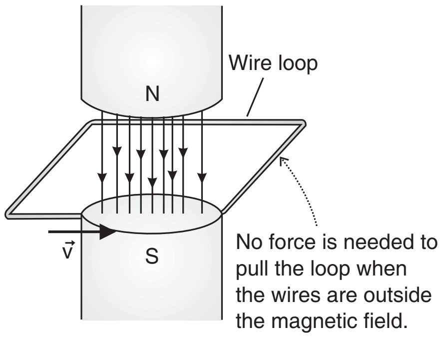
TOPIC-2

Eddy Currents, Self and Mutual Induction

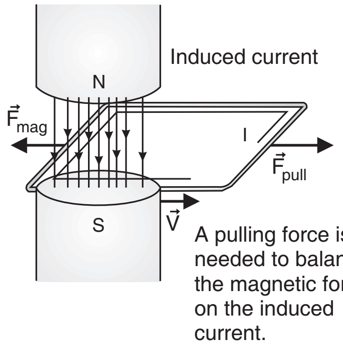
Quick Review

Eddy Currents

- Current loops induced in moving conductors are called eddy currents. They can create significant drag, called as has magnetic damping.
- Eddy currents give rise to magnetic fields that oppose any external change in the magnetic field.
- It is written as $i = \dfrac{e}{R}$
- Eddy currents are induced electric currents that flow in a circular path



No force is needed to pull the loop when the wires are outside the magnetic field.



A pulling force is needed to balance the magnetic force on the induced current.

- Eddy currents flowing in a material will generate their own secondary magnetic field that opposes the coil's primary magnetic field.

Mutual Induction :

- The production of induced emf in a circuit, when the current in the neighbouring circuit changes is called mutual induction.

  When the circuit of the primary coil is closed or opened, deflection is produced in the galvanometer of the secondary coil. This is due to the mutual induction.
- The mutual induction between two coils depends on the following factors :
  - The number of turns of primary and secondary coils.
  - The shape, size or geometry of the two coils. That is the area of cross-section and the length of the coils affect the mutual induction between the coils.

Coefficient of mutual induction :

- Suppose, the instantaneous current in the primary coil is I. Let the magnetic flux linked with the secondary coil be ϕ. It is found that the magnetic flux is proportional to the current i.e.,

$$\phi \propto I \text{ or } \phi = MI \quad ...(i)$$

where, M is the constant of proportionality. It is called coefficient of mutual induction.

The induced emf ε in the secondary coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -M\frac{dI}{dt} \qquad ...(ii)$$

The negative sign is in accordance with the Lenz's law. That is, *the induced emf in the secondary coil opposes the variation of current in the primary coil.*

From the equation (ii), we find

$$M = \frac{\varepsilon}{(dI/dt)}$$

Now $$M = \frac{\varepsilon}{dI/dt}.$$

Therefore, $$\text{Unit of } M = \frac{V}{As^{-1}} = VA^{-1}s$$

Self-Induction :

- The production of induced emf in a circuit, when the current in the same circuit changes is known as **self-induction**.

  Suppose the instantaneous current in the circuit is I and if the magnetic flux linked with the solenoid is f, then it is found that :

  $$\phi \propto \phi \text{ or } \phi = LI \qquad ...(i)$$

  where, L is the constant of proportionality. It is called coefficient of self-induction.

  The induced emf in the coil is given by

  $$\varepsilon = \frac{d\phi}{dt} = -L\frac{dI}{dt} \qquad ...(ii)$$

  The negative sign is in accordance with the Lenz's law. That is, *the induced emf opposes the variation of current in the coil.*

  From the equation (ii), we find :

  $$L = \varepsilon/(dI/dt) \qquad ...(iii)$$

  Then, the coefficient of self-induction is the ratio of induced emf in the circuit to the rate of change of the current in the circuit.

  Unit of L : The unit of self-induction is also called henry (symbol H).

  From equation (ii), we find that if $dI/dt = 1$

  $\varepsilon = 1\,V,$

  then $L = 1\,H \Rightarrow \text{Unit of } H = VA^{-1}s$
- If a rod of length l moves perpendicular to a magnetic field B with a velocity v, then the induced emf produced across it, is given by

  $$\varepsilon = vBl$$

  In general, we have, $$\varepsilon = \vec{B}.(\vec{v}\times\vec{l})$$
- If a metallic rod of length l rotates about one of its ends in a plane perpendicular to the magnetic field, then the induced emf produced across its ends is given by

  $$\varepsilon = \frac{B\omega l^2}{2} = \frac{B2\pi f l^2}{2} = BAf$$

  Here, w = angular velocity of rotation, A = pl^2 = area of circle and f = frequency of rotation.
- Lenz's law explains the direction of induced emf.
- Inductance in the electrical circuit is equivalent to the inertia (mass) in mechanics.
- When a bar magnet is dropped into a coil, the electromagnetic induction in the coil opposes its motion, so the magnet falls with acceleration less than that due to gravity.
- The inductance of a coil depends on the following factors :
  - area of cross-section,
  - number of turns
  - permeability of the core.
- Unit of induction, $H = \frac{Wb}{A} = \frac{Vs}{A} = \Omega.s$
- The inductance of a circular coil is given by :

  $$L = \frac{\phi}{I} = \frac{BAN}{I} = \frac{\mu}{4\pi}.\frac{(2\pi NI)}{rI}\times AN \qquad \left[\because B = \frac{\mu}{4\pi}.\frac{2\pi NI}{r}\right]$$

$$L = \frac{\mu N^2}{2r}A = \frac{\mu N^2}{2r} \times \pi r^2$$

or $$L = \frac{\mu N^2 \pi r}{2}$$

Here,f = magnetic flux from the coil, I = current through the coil, A = area of coil, r = radius of coil, N = total number of turns of the coil, m = permeability of the medium.

- The inductance of a solenoid of length l is given by

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \left(\frac{\mu NI}{l}\right)\frac{AN}{I} \qquad \left[\because B = \frac{\mu NI}{l}\right]$$

or $$L = \frac{\mu N^2 A}{l} = \mu n^2 Al = \mu n^2 V \qquad \left[\because n = \frac{N}{l}\right]$$

Here, $n = N/l$ = number of turns per unit length and $V = Al$ = volume of the solenoid.

- If two coils of inductance L_1 and L_2 are coupled together, then their mutual inductance is given by

$$M = k\sqrt{L_1 L_2}$$

where, k is called the coupling constant.

- The value of k lies between 0 and 1.

  For perfectly coupled coils, $k = 1$, it means that the magnetic flux of primary coil is completely linked with the secondary coil.
- Eddy currents do not cause sparking.
- If a current I is set up in a coil of inductance L, then the magnetic field energy stored in it is given by

$$U_m = \frac{1}{2}LI^2$$

Know the Terms

- **Back emf :** Emf generated by a running motor due to coil that turns in a magnetic field which opposes the voltage that powers the motor.
- **Eddy current :** Current loop in a conductor that is caused by motional emf.
- **Mutual induction :** It is a change of flux in one element that induces an emf in other.
- **Transformer :** It is a device that increases or decreases an *ac* voltage.
- **Inductor :** It is a device used to store electrical energy in a magnetic field when electric current flows.

Know the Formulae

- $$\varepsilon = \frac{d\phi}{dt} = -L\frac{dI}{dt}$$
- $$\varepsilon = \frac{d\phi}{dt} = -M\frac{dI}{dt}$$
- The inductance in series is given by

$$L_s = L_1 + L_2 + L_3 + ...$$

- The inductance in parallel is given by

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ...$$

- Mutual Inductance of two coils is given by

$$M = \frac{\mu_0 \mu_r N_P N_S A_P}{I_P} = \frac{\mu_0 \mu_r N_P N_S A_S}{I_P}$$

where, μ_0 is the permeability of fee space ($4\pi \times 10^{-7}$)

μ_r is the relative permeability of the soft iron core

N_S is number of turns is secondary coil.

N_P is number of turns is primary coil.

A_P is the cross-sectional area of primary coil in m^2.

A_S is the cross-sectional area of secondary will in m^2.

I is the coil current.

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Chapter - 7 : Alternating Currents

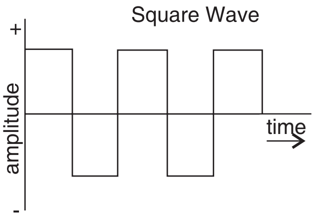
TOPIC-1

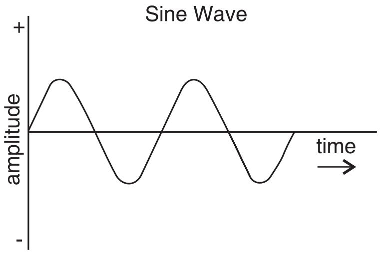
Alternating Current

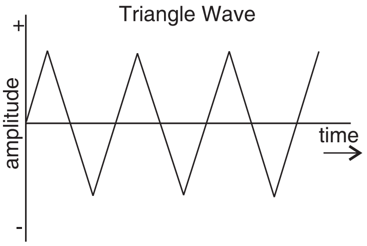
Quick Review

Alternating currents

- Alternating current is that which changes continuously in magnitude and periodically in direction.
- It is represented by sine curve or cosine curve as $I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$ where I_0 is peak value of current and I is instantaneous value of current.
- Frequency of an alternating current supply f, is the number of cycles which gets completed per second which is measured in Hertz (Hz).
- The time period T, of an alternating supply is time taken to complete one cycle.
- The behaviour of ohmic resistance R in *ac* circuit is the same as in *dc* circuit.
- The alternating EMF and alternating current are in same phase.
- Alternating current can be produced using a device called an alternator.
- *ac* **waveforms are :**







Peak and *rms* **value of alternating current/voltage**

- Root mean square or *rms* is the root of mean of square of voltage or current in an *ac* circuit for one complete cycle denoted by V_{rms} or I_{rms}
- *rms* value is the standard way of measuring alternating current and voltage as it gives the *dc* equivalent values.
- *rms* value of *ac* is also called effective value or virtual value of *ac* represented as I_{rms}, I_{eff} or I_v shown as

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

- *rms* voltage value is the square root of averages of the squares of instantaneous voltages in a time varying waveform.

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

Reactance and impedance

- When an *ac* current is passed through a resistance, a voltage drop is produced which is in phase with the current and is measured in ohms (Ω).
- Reactance is the inertia against the motion of electrons where an alternating current after passing through it produces a voltage drop which is 90° out of phase with the current.
- Reactance is shown by "*X*" and is measured in ohms (Ω).
- Reactance is of two types; inductive and capacitive.
- Inductive reactance is linked with varying magnetic field that surrounds a wire or a coil carrying a current.
- Inductive reactance (X_L) is the resistance offered by an inductor and is given by $X_L = \omega L = 2\pi f L$
- Through a pure inductor, alternating current lags behind the alternating *emf* by phase angle of 90°.
- Capacitive reactance is linked with changing electric field between two conducting surfaces separated from each other by an insulating medium.
- Capacitive reactance (X_c) is the resistance offered by a capacitor and is given by

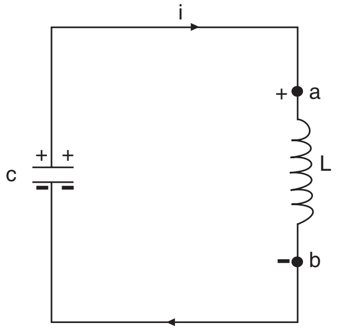
$$X_C = \frac{1}{2\pi f C}$$

- Through a pure capacitor, alternating current leads the alternating *emf* by a phase angle of 90^o.
- Impedance is the comprehensive expression of all forms of opposition to electron flow, including resistance and reactance, where an alternating current after passing through it produces a voltage drop between 0^o and 90^o which will be out of phase with current given as,

$$Z = \sqrt{R^2 + X^2}$$

where,
Z = impedance of circuit
R = resistance
X = reactance

LC oscillations (qualitative treatment only)



- LC circuit comprises of inductor and capacitor connected in series where energy from the cell is given to capacitor which will keep on oscillating among L & C.
- When *ac* voltage is applied to the capacitor, it will keep on charging and discharging continuously.
- When capacitor is fully charged, it starts discharging and charge gets transferred to the inductor which is connected to capacitor.
- Due to change in current, there will be change in magnetic flux of the inductor in the circuit which induces an emf in the inductor.\
- The *emf* is given by $e = -L\frac{dI}{dt}$ which try to oppose the growth of the current.
- When capacitor gets completely discharged, all the energy stored in it will now store in an inductor as a result of which, inductor will start charging the capacitor and energy stored in the capacitor will start increasing.
- As there is no current in the circuit, energy in the inductor is zero, so total energy of LC circuit will be

$$U_E = \frac{1}{2}.\frac{q^2}{C}$$

Know the Terms

- **AC current :** It is a current that fluctuates sinusoidal with time at fixed frequency.
- **AC voltage :** It is the voltage that fluctuates sinusoidal with time at fixed frequency.
- **Alternating current :** It is the flow of electric charge that periodically reverses direction.
- **Bandwidth :** It is the range of angular frequencies over which the average power is greater than 1/2 the maximum value of average power.
- **Capacitive reactance :** It is the opposition of a capacitor to a change in current.
- **Direct current :** It is the flow of electric charge in only one direction.
- **Impedance :** Is an *ac* analog to resistance in a *dc* circuit that measures the combined effect of resistance, capacitive reactance, and inductive reactance.
- **Inductive reactance :** It is the opposition of an inductor to a change in current.

Know the Formulae

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$\text{Peak Power} = V_0 I_0$$

$$\text{Average Power} = \frac{1}{2}V_0 I_0 \cos\phi = V_{rms} I_{rms}\cos\phi \qquad \text{where, } \cos\phi = \frac{R}{Z} \text{ is power factor}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$X_L = \omega L = 2\pi f L$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$emf_e = -L\frac{dI}{dt}$$

$$\text{Energy in } LC \text{ circuit, } U_E = \frac{1}{2}\frac{q^2}{C}$$

TOPIC-2

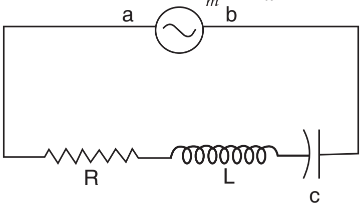
LCR Series Circuit

Quick Review

LCR series circuit

- In an *LCR* series circuit with resistor, inductor and capacitor, the expression for the instantaneous potential difference between the terminals *a* and *b* is given as

$$V = V_m \sin \omega t$$



- The potential difference in this will be equal to the sum of the potential differences across *R*, *L* and *C* elements as

$$V_m \sin\omega t = RI + L\frac{di}{dt} + \frac{1}{C}q$$

where, q is the charge on capacitor.

- The expression for a steady-state current and for transient current results as :

$$\omega V_m \cos\omega t = L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i$$

- The steady state solution will be

$$i = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\sin(\omega t - \phi)$$

$$\text{where, } \phi = \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R}$$

- From the equation, steady-state current like terminal voltage, varies sinusoidal with time, so steady-state current can be written as $I = I_m \sin(\omega t - \varphi)$
- **In an *LCR* circuit:**

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + X^2}$$

$$\mathrm{I_m} = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_m}{\sqrt{R^2 + X^2}} = \frac{V_m}{Z}$$

Here,
Z = impedance of the circuit
X = reactance
X_L and X_C = inductive and capacitive reactance

- For steady-state currents, maximum current I_m is related to maximum potential difference V_m by

$$I_m = \frac{V_m}{Z}$$

- Total effective resistance of *RLC* circuit is called Impedance (Z) of the circuit given as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- The angle by which alternating voltage leads the alternating current in *RLC* circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R}$$

- In an *RLC* circuit, impedance triangle is a right-angled triangle in which base is ohmic resistance R, perpendicular is reactance ($X_L - X_C$) and hypotenuse is impedance (Z)
- When a condenser of capacity C charged to certain potential is connected to inductor L, energy stored in C oscillates between L and C where frequency of energy oscillations is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

- In *LCR* circuit, if there is no loss of energy, then total energy in L and C at every instant will remain constant.
- Sign for phase difference (ϕ) between I and E for a series *LCR* circuit :
  ϕ is positive, when $X_L > X_C$.
  ϕ is negative, when $X_L < X_C$.
  ϕ is zero, when $X_L = X_C$.
  $\phi = \pi/2$, when $\omega = \infty$.
  $\phi = -\pi/2$, when $\omega = 0$.
- The maximum current in an *LR* circuit is determined by the *emf* E and resistance R.
- The maximum charge on the capacitor in a *RC* circuit is determined by the *emf* E and capacitance C.
- A capacitor cannot be charged or discharged instantaneously.
- The dimensions of L/R are same as those of time period.
- The dimensions of R/L are same as that of frequency.
- The dimensions of RC are same as that of time period.

  Dimension of $\frac{L}{R}$, RC, $\sqrt{LC}$ = Time

Resonance

- Circuit in which inductance L, capacitance C and resistance R are connected in series and the circuit admits maximum current, then such circuit is called as series resonant circuit.
- The necessary condition for resonance in *LCR* series circuit is :

  $X_L = X_C$ which gives $$f = \frac{1}{2\pi\sqrt{LC}}$$

- In this, frequency of *ac* fed to circuit will be equal to natural frequency of energy oscillations in the circuit under conditions,

$$Z = R$$

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$$

- In an *LCR* circuit, if voltage and current are in phase, then necessary condition will be:

$$X_L = X_C$$

since $$\tan\phi = \frac{X_L - X_C}{R}$$

and if $$X_L = X_C$$

then $$\phi = 0,\ \omega = 2\pi f,$$

Also, $$X_L = \omega L = 2\pi f L$$

and $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C},$$

so frequency will be
$$f=\frac{1}{2\pi\sqrt{LC}}$$

- The sharpness of turning at resonance is measured by Q factor or quality factor of the circuit given as
$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}$$
- At series LCR resonance or acceptor circuit, current is maximum given as
$$I_{\max}=\frac{E}{R}$$
- At parallel LCR resonance or rejecter circuit, current is minimum given as
$$I_{\min}=\frac{E}{R}$$

Power in AC circuits

- Power is the rate at which work is performed or energy is supplied.
- In the case of direct current, the rate at which a circuit supplies energy to an electrical device is given by
$$P = VI$$
where,
V = potential difference between the terminals of the device
I = current flowing through the device
- When the current is out of phase with the voltage, the power indicated by the product of the applied voltage and the total current gives only what is known as apparent power.
- Instantaneous power is the time function of power,
$$p(t) = u(t) \times i(t)$$
which is the product of time functions of the voltage and current.
- Complex power is the product of the complex effective voltage and the complex effective conjugate current given as
$$S=U\times I=\frac{U_M\times I_M}{2}$$
- Real or average power defined as P can be defined as real part of complex power or simple average of instantaneous power given as
$$P=\frac{1}{T}\int_{t_0}^{t_0+T}u(t)\times i(t)dt$$
- Reactive power (Q) is the imaginary part of complex power measured in volt-amperes reactive (VAR) which is positive in inductive circuit while negative in capacitive circuit.
$$Q=(I_X)^2X=(I_L)^2X_L-(I_C)^2X_C$$
where,
I_X = reactive current in amperes, X= total reactance, I_C = capacitive current
X_C = capacitive reactance, I_L = inductive current, X_L= inductive reactance
- Apparent power is the power which appears to the source due to circuit impedance given as :
$$S=(I_Z)^2Z$$
where, I_Z=impedance current, Z = impedance
- Apparent power is the combination of true power and reactive power which is measured in VA.
$$\text{S}=\sqrt{(\text{True power})^2+(\text{Reactive power})^2}$$
- Power factor (cos ϕ) is important in power systems as it shows how closely the effective power equals the apparent power given as :
$$\cos\phi=\frac{\text{effective power}}{\text{apparent power}}$$
- The value of power factor varies from 0 to 1.
- The instantaneous rate at which energy is supplied to an electrical device by *ac* circuit is
$$P = VI$$
- Average power in RLC where, $X_L = X_C$ over a complete cycle in a non inductive circuit or pure resistive circuit is given as $P = V_0I_0$ or I_0^2R
- In an LCR circuit, average power associated over a complete cycle is $P = V_{rms}I_{rms}\cos\theta$, where

- ϕ = phase angle between alternating voltage and alternating current

$$\cos\phi = \frac{\text{effective power}}{\text{apparent power}}$$

Wattless Current

- The average power associated over a complete cycle with pure inductor or pure capacitor is zero which makes current through *L* and *C* as wattless or idle current.
- In LCR circuit at resonance, the power loss is maximum, so
- Wattless component of current = $I_{rms} \sin\phi$
- Power component of current = $I_{rms} \cos\phi$

Know the Terms

- **Phase angle :** It is the amount by which the voltage and current are out of phase with each other in a circuit.
- **Power factor :** It is the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase.
- **Quality factor :** It is a dimensionless quantity that shows sharpness of the peak of bandwidth.
- **Resonant frequency :** It is the frequency at which the amplitude of the current is maximum where circuit oscillate when not driven by voltage source.

Know the Formulae

- Impedance for a series LCR circuit,

$$Z = \sqrt{R^2+X^2} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}.$$

- Admittance for a parallel LCR circuit,

$$Y = \frac{1}{Z} = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{-\frac{1}{2}}$$

- Average power, $P = \dfrac{E_0 I_0}{2} \cos\phi$
- Power factor, $PF = \cos\phi = \dfrac{\text{Resistance}}{\text{Impedance}}$

$$= \frac{\text{True power}}{\text{Apparent power}}$$

TOPIC-3

Generator and Transformer

Quick Review

ac generator

- An alternator is an electrical machine which converts mechanical energy into alternating electric energy.
- Alternator or a synchronous generator has a stator and rotor.
- It is similar to basic working principle of a *dc* generator.
- It works on the principle of electromagnetic induction where a coil gets rotated in uniform magnetic field, sets an induced *emf* given as :

$$e = e_0 \sin\omega t = NBA\omega \sin \omega t$$

Transformer

- Transformer is an electrical device used for changing the alternating voltages.
- It is based on the phenomenon of mutual induction.
- The main use of transformer is in transmission of *ac* over long distances at extremely high voltages which reduces the energy losses in transmission.
- It comprises of two sets of coils which are insulated from each other and are wound on soft-iron core.

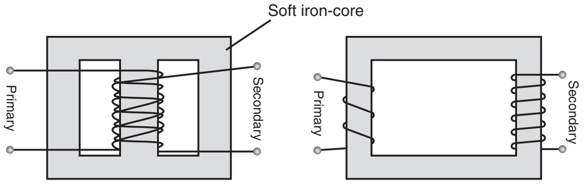
- In this, one of the coil is called as primary (input coil) having N_p turns while other coil is secondary (output coil) having N_s turns, so we have

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- **Efficiency of transformer:**

$$\eta = \frac{\text{output power}}{\text{input power}}$$

$$\eta = \frac{E_s I_s}{E_p I_p}$$



- Inspite of heavy power losses, the efficiency in a transformer is usually above 90%.
- An ideal transformer is 100% efficient as it delivers all energy it receives.
- Real transformer is not 100% efficient and at full load, its efficiency lies between 94% to 96%.
- A transformer operating with constant voltage and frequency with very high capacity, efficiency results as 98%.
- **Transformer Ratio :**

$$E_s = \left(\frac{N_s}{N_p}\right) E_p \; and \; I_s = \left(\frac{N_p}{N_s}\right) I_p$$

$\frac{N_s}{N_p} = \frac{V_s}{V_p}$ is defined as the transformer ratio.

The value of turns ratio of a transformer $\frac{N_p}{N_s} = \frac{V_p}{V_s} = n$

- **Step-up transformer :** If secondary coil has more number of turns than primary ($N_s > N_p$), voltage gets stepped up ($V_s > V_p$).

In this, there is less current in secondary as compared to primary ($\frac{N_s}{N_p} < 1$ and $I_s < I_p$).

The value of transformer ratio $k > 1$

- **Step-down transformer :** In this, the secondary coil has less number of turns than primary ($N_s < N_p$). In this, $V_s < V_p$ and $I_s > I_p$ as voltage gets stepped down or reduced with increase in current.

In this, value of transformer ratio $k < 1$

- The main use of transformers is in stepping up voltage for power transmission.
- Electric power can be transmitted much more efficiently at high voltages than at low voltages due to less I^2R heat loss in a high voltage / low current transmission.
- **Energy losses in transformers :**

1. Flux Leakage
2. Resistance of windings
3. Eddy currents
4. Hysteresis

Know the Terms

- **Mutual induction :** It is a process by which a coil of wire magnetically induces a voltage in other coil placed in close proximity to it.

Know the Formulae

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

$$\eta = \frac{E_s I_s}{E_p I_p}$$

$$\%\text{efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

$$= \frac{\text{Input power } - \text{ Losses}}{\text{Input power}} \times 100\%$$

$$V_s = \left(\frac{N_s}{N_p}\right) V_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right) I_p$$

- $e = e_0 \sin \omega t = NBA\omega \sin \omega t$
- $I = e/r \quad I = \dfrac{NBA\omega \sin \omega t}{R}$
- The value of transformer ratio is greater than 1 for step up transformer and less than 1 for step-down transformer.`

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UNIT - V : Electromagnetic Waves

Chapter - 8 : Electromagnetic Waves

TOPIC-1

Displacement Current

Quick Review

Basic idea of displacement current :

- Displacement current is a quantity appearing in Maxwell's equations that is defined in terms of the rate of change of electric displacement field (*D*).
- Displacement current density has the units of electric current density and has associated magnetic field similar as actual currents.
- Displacement currents play an important role in the propagation of electromagnetic radiation, such as light and radio waves, through empty space.
- Maxwell in its equation, added an additional term which includes a factor known as displacement current, I_d.
- Displacement current is defined as :

$$I_d = \varepsilon_0 \frac{d\phi_\varepsilon}{dt},$$

where $\phi_E = \int E.dA$ is the electric flux and ε_0 is the permittivity of free space.

- Gauss's law shows that total electric flux through a Gaussian surface is equal to the charge enclosed by it,

$\oint_s \varepsilon_0 \vec{E}.d\vec{S} = \oint_s \vec{D}.\,dS = q$ where $\vec{D} = \varepsilon_0.\vec{E}$ is displacement vector.

- As per Ampere Maxwell law, line integral of magnetic field around any closed path is equal to μ_0 × (sum of conduction current and displacement current through that path.)

i.e.,

$$\oint \vec{B}.\vec{dl} = \mu_0[I_c + I_d]$$

$$I_d = \frac{dq}{dt} = \frac{\varepsilon_0 d\phi_E}{dt}$$

$$\oint \vec{B}.\vec{dl} = \left[L_c + \frac{\varepsilon_0 d\phi_E}{dt}\right]$$

or

$$\oint \vec{B}.\vec{dl} = \mu_0 I_c + \frac{\mu_0 \varepsilon_0 d\phi_\varepsilon}{dt}$$

Electromagnetic waves and their characteristics (qualitative ideas only)

- EM waves are produced by accelerated charge.

- The electric and magnetic fields produced by accelerated charge changes with time, which radiates electromagnetic waves.

Example :

- Electron jumping from its outer to inner orbits radiates EM waves.
- Electrical oscillations in LC circuit produces EM waves.
- Electric sparking generates EM waves.

Characteristics of EM waves :

- EM waves are propagated as electric and magnetic fields oscillating in mutually perpendicular directions.
- EM waves travel in vacuum along a straight line with the velocity 2.997924591×10^8 m/s which is often assumed as 3×10^8 m/s.
- EM waves are not affected by electric and magnetic fields.
- Relation between electric and magnetic field components is

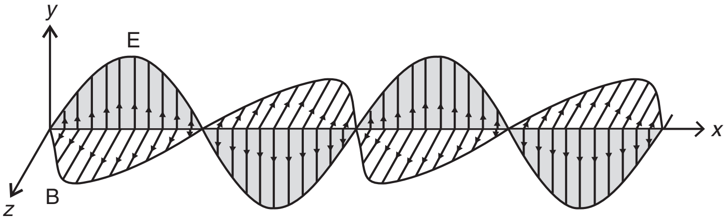
$$E = Bc,$$

where, $c \cong 3 \times 10^8$ m/s.

- Electromagnetic waves can be of wave length (λ) from 0 to ∞. Also, corresponding frequencies (f) can be from ∞ to 0. The λ and f are related as $c = f\lambda$.

Transverse nature of electromagnetic waves

- In electromagnetic wave, electric and magnetic field vectors are perpendicular to each other in the direction of propagation of wave which shows its transverse nature.



- A plane EM wave traveling in the x-direction is of the form:

$$E(x, t) = E_{max}\cos(kx - \omega t + \phi)$$
$$B(x, t) = B_{max}\cos(kx - \omega t + \phi)$$

where,

E = electric field vector
B = magnetic field vector

- In this, as wave propagates along Z-axis, the electric and magnetic field propagation will be:

$$E = E_0\sin(kz - \omega t)$$
$$B = B_0\sin(kz - \omega t)$$

Know the Terms

- **Displacement current :** A quantity in Maxwell's equations describing in terms of rate of change of electric displacement field.
- **Gauss's Law :** It shows total electric flux out of closed surface equals to the charge enclosed divided by permittivity.
- **Electromagnetic waves :** Waves that can travel through the vacuum of outer space and needs the presence of material medium for transporting energy from one location to another.

Know the Formulae

- Displacement current between the plates of a capacitor

$$I_D = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

$$I_D = \varepsilon_0 A \frac{d}{dt}(V/d) = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

Here, E = electric field between the plates of the capacitor, V = potential difference, d = separation between the plates, C = capacitance of the capacitor, A = area of plates.

- For the EM waves, the energy density is given by

$$U_E = \frac{1}{2}\varepsilon_0 E^2 \text{ (Due to electric field)}$$

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0} \text{ (Due to magnetic field)}$$

- The energy transported by EM waves per unit area per second is called Poynting vector ($\vec{S}$)

  It is given by $$\vec{S} = \vec{E} \times \frac{\vec{B}}{\mu_0}.$$

  Since, $\vec{E} \perp \vec{B}$, hence $$S = \frac{EB}{\mu_0}.$$

- In EM waves, the half of the energy is electric and the other half is magnetic, *i.e.*, the total energy density of EM waves is

$$u = \varepsilon_0 E^2 + \frac{B^2}{\mu_0}$$

- $$\frac{1}{\mu_0 \varepsilon_0} = c^2$$
- The variation in magnetic field causes electric field and vice versa.
- In the EM waves $\vec{E} \perp \vec{B}$, both $\vec{E}$ and $\vec{B}$ are in the same phase.
- In the EM waves : $E = E_0 \sin(\omega t - kx)$, $B = B_0 \sin(\omega t - kx)$.
- The EM waves travel in the direction of $\vec{E} \times \vec{B}$ *i.e.*, EM waves propagate perpendicular to both $\vec{E}$ and $\vec{B}$.

Maxwell's Equation :

1. $\oint E.dA = \frac{Q}{\varepsilon_0}$ (Gauss's Law for electricity).

2. $\oint B.dA = 0$ (Gauss's Law for magnetism).

3. $\oint E.dl = -\frac{d\phi_B}{dt}$ (Faraday's law).

4. $\oint B.dl = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$ (Ampere-Maxwell Law).

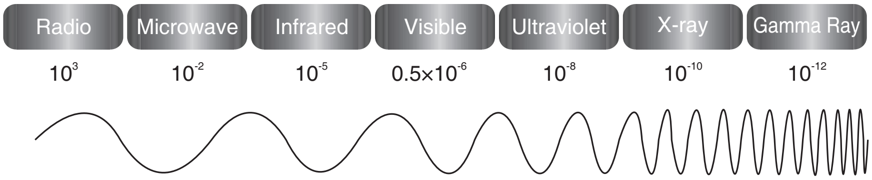
TOPIC-2

Electromagnetic Spectrum

Quick Review

Electromagnetic spectrum

- Electromagnetic spectrum describes all wavelengths of light from dark nebulae to stars revealing an invisible universe.
- It describes the entire range of light from radio waves to gamma rays.

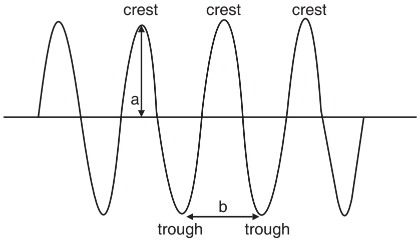


- In EM spectrum, the electromagnetic radiation ranges from wavelengths between 10^{-14} m and 10^{15} m.

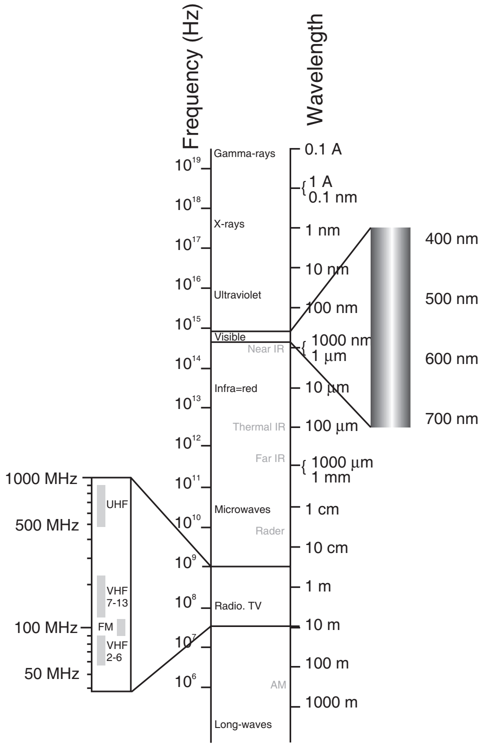
Types of Electromagnetic waves (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays)

- Electromagnetic waves require no medium to travel or propagate.
- Electric and magnetic fields produce vibrations and creates electromagnetic waves.
- Electromagnetic waves are transverse waves which are measured by their height, amplitude, wavelength, or distance between highest points of one wave with the other.

- In electromagnetic waves, distance between the highest point of wave is known as crest while the lowest point of wave is known as trough.



- EM radiations are classified as per the frequency of wave such as radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.



Electromagnetic spectrum is divided into following regions :

Gamma-rays

[wavelength 10^{-14} m – 10^{-11} m, frequency 10^{22} Hz - 10^{19} Hz, mean energy per quantum 6.6×10^{-14} J = 4×10^{5} eV]

X-rays

[wavelength 10^{-12} m –10^{-8} m, frequency 10^{20} Hz - 10^{16} Hz, mean energy per quantum 6.6×10^{-17} J = 4×10^{2} eV]

Ultraviolet radiation

[wavelength 10^{-8} m – 10^{-6} m, frequency 10^{17} Hz - 10^{15} Hz, mean energy per quantum 6.6×10^{-20} J = 4×10^{-1} eV]

Visible light

[wavelength 10^{-7} m - 10^{-6} m, frequency 10^{15} Hz - 10^{14} Hz, mean energy per quantum 6.6×10^{-19} J = 4×10^{-2} eV]

Infrared radiation

[wavelength 10^{-6} m - 10^{-3} m, frequency 10^{14} Hz - 10^{12} Hz, mean energy per quantum 6.6×10^{-21} J = 4×10^{-4} eV]

Microwaves

[wavelength 10^{-4} m - 10^{-1} m, frequency 10^{13} Hz - 10^{9} Hz, mean energy per quantum 6.6×10^{-23} J = 4×10^{-6} eV]

Radio waves

[wavelength 10 m - 10^{3} m, frequency 10^{8} Hz - 10^{6} Hz, mean energy per quantum 6.6×10^{-26} J = 4×10^{-9} eV]

Uses of Electromagnetic waves

Band Designation	Frequency (Hz)	Wavelength	Applications
Audible	20 Hz - 20 kHz	>100 km	Acoustics
Extremely Low Frequency (ELF) Radio	30 Hz - 300 Hz	10,000 km - 1,000 km	Electronics, Submarine Communications
Infra Low Frequency (ILF)	300 Hz – 3 kHz	1,000 km - 100 km	Not Applicable
Very Low Frequency (VLF) Radio	3 kHz - 30 kHz	100 km - 10 km	Navigation, Weather
Low Frequency (LF) Radio	30 kHz - 300 kHz	10 km - 1 km	Navigation, Maritime Communications, Information and Weather Systems, Time Systems
Medium Frequency (MF) Radio	300 kHz - 3 MHz	1 km – 100 m	Navigation, AM Radio, Mobile Radio
High Frequency (HF) Radio	3 MHz – 30 MHz	100 – 10 m	Citizens Band Radio, Mobile Radio, Maritime Radio
Very High Frequency (VHF) Radio	30 MHz -300 MHz	10 m - 1 m	Amateur (Ham) Radio, VHF TV, FM Radio, Mobile Satellite, Mobile Radio, Fixed Radio
Ultra High Frequency (UHF) Radio	300 MHz - 3 GHz	1 m - 10 cm	Microwave, Satellite, UHF TV, Paging, Cordless Telephone, Cellular and PCS Telephony, Wireless LAN (WiFi)
Super High Frequency (SHF) Radio	3 GHz - 30 GHz	10 cm – 1 cm	Microwave, Satellite, Wireless LAN (WiFi)
Extremely High Frequency (EHF) Radio	30 GHz - 300 GHz	1 cm - 1 mm	Microwave, Satellite, Radiolocation
Infrared Light (IR)	300 GHz – 400 THz	1 mm - 750 nm	Wireless LAN Bridges, Wireless LANs, Fiber Optics
Visible Light	400 THz – 1 PHz	750 nm - 380 nm	Not Applicable
Ultraviolet (UV)	1 PHz – 30 PHz	380 nm -10 nm	Not Applicable
X-Rays	30 PHz - 30 EHz	10 nm - .01 nm	Not Applicable
Gamma and Cosmic Rays	>30 EHz	<0.01 nm	Not Applicable

Know the Terms

- **Electromagnetic waves :** Waves that appears from changing of electric and magnetic fields.
- **Gamma rays :** Rays with smallest wavelengths and highest frequencies having high energy capable of travelling long distances through air and are most penetrating.
- **X-rays :** These are the rays with long and small wavelengths having higher energy as compared to ultraviolet radiation.
- **Ultraviolet (UV) radiation :** It is a part of electromagnetic spectrum that lies between X-rays and visible light.
- **Visible light :** It is a visible spectrum which is part of electromagnetic spectrum which can be seen by human eyes.
- **Infrared (IR) radiation :** These are thermal radiation which is part of electromagnetic spectrum that lies between visible light and microwaves.
- **Radio waves :** Waves with long wavelengths used in television, cell phone and radio communications.
- **Wavelength :** It is the distance between one wave crest to the next.

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UNIT - VI : Optics

Chapter - 9 : Ray Optics and Optical Instruments

TOPIC-1

Reflection of Light Rays by Spherical Mirrors

Quick Review

- Light is a form of energy. Ray of light represents direction of propagation of light energy.
- The speed of light in vacuum is the highest speed attainable in nature. Its approx. value is 3×10^8 m/s.
- When light falls on any object/surface, there are three optical phenomenon which occur, reflection, refraction and absorption of light by the object/ surface.
- By law of conservation of energy, sum of reflected, absorbed and transmitted light is always equal to the incident light.
- Depending upon the amount of light it reflects, transmits or absorbs object is classified into good reflector, transmitter or absorber of light.

Reflection of Light by Spherical Mirror

- When light falls on an object, it bounces back the light in the same medium from where the light comes. This is called the reflection of light.
- Mirrors are good reflectors. A mirror can be made by silvering a metal surface with glass in front and paint at its back.

  Laws of Reflection : It is observed that light follows the following laws while reflecting from any type of surfaces.

  (i) The angle of incidence is equal to the angle of reflection, and

  (ii) The incident ray, the normal to the mirror at the point of incidence and the reflected ray, all lie in the same plane.

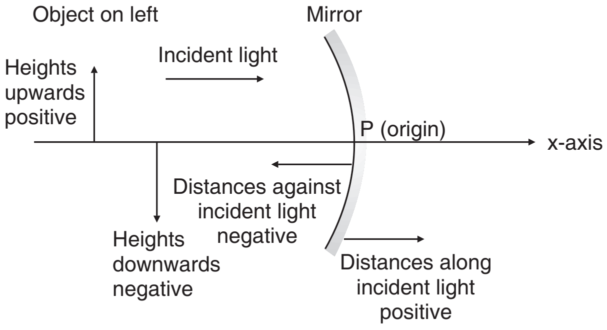
Spherical Mirror

- Curved shaped mirrors are known as spherical mirrors. Depending upon the type of curve of reflecting surface, spherical mirrors are categorized as :
  - **Concave Mirror :** A spherical mirror, whose reflecting surface is curved inwards is called a concave mirror. It means reflecting (polished) surface faces the center of the sphere from which it is made.
  - **Convex Mirror :** A spherical mirror whose reflecting surface is curved outwards is called a convex mirror.

Important Terms related to spherical mirrors

- The mid point or the centre of the reflecting surface of the mirror is known as pole of the mirror. It is represented by P.
- The centre of the hollow sphere from which the mirror is made is known as centre of curvature. It is represented by C. Centre of curvature in concave mirror is in front of the mirror and in convex mirror, it is behind the mirror.
- An imaginary straight line which joins Pole and Centre of curvature of the mirror is known as principal axis and the distance between the Centre of curvature and Pole of the mirror is called the radius of curvature. It is represented by R.
- For mirrors whose radius of curvature is much larger than aperture, there will be relation between R and f such that $f = R/2$.
- Image is perception of object. If rays emanating from a point actually meet at another point then that point is real image of the point. The image will be virtual if the rays do not actually meet but appear to diverge from the point when produced backward.

The Cartesian Sign Convention



- **Image formation in concave mirror for different position of object :**

Position of the object	Position of the image	Size of the image	Nature of the image
At infinity	At the focus *F*	Highly diminished, point sized	Real and inverted
Beyond *C*	Between *F* and *C*	Diminished	Real and inverted
At *C*	At *C*	Same size	Real and inverted
Between *C* and *F*	Beyond *C*	Enlarged	Real and inverted
At *F*	At infinity	Highly Enlarged	Real and inverted
Between *P* and *F*	Behind the mirror	Enlarged	Virtual and erect

- **Image formation in convex mirror for different position of object :**

Position of the object	Position of the image	Size of the image	Nature of the image
At infinity	At the focus *F*, behind the mirror	Highly diminished point sized	Virtual and Erect
Between infinity and the pole *P* of the mirror	Between *P* and *F*, behind the mirror	Diminished	Virtual and Erect

- **Mirror Formula :** In a spherical mirror, there is a relation between object's distance u, image distance v and principal focus of the mirror f.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

- **Magnification by Mirror :** The extent by which mirror extends or reduces the size of image with respect to object is called the magnification factor of mirror. It is represented by m. If size of an object is h and its image by spherical mirror is h'. Then magnification factor of mirror is

$$m=-\frac{v}{u}=\frac{h'}{h}$$

TOPIC-2

Refraction of Light through-Glass Slab, Prism & Lenses and Total Internal Reflection

Quick Review

- **Refraction of light :** Refraction is bending of light when it obliquely travels from one medium to another medium. **Snell experimentally found the following law of refractions.**

Laws of Refraction of Light :

- The incident ray, the refracted ray and the normal to the interface of two transparent media at the point of incidence, all lie in the same plane.
- The ratio of sine of angle of incidence to the sine of angle of refraction is a constant, for the light of a given colour and for the given pair of media. This constant value is called the refractive index of the second medium with respect to the first medium.

$$\frac{\sin i}{\sin r}=\text{constant }(n_{21})$$

This is known as Snell's law.

- From Snell's law

$$\sin i = \sin r \times n_{21}.$$

It shows that if $\angle i = 0$ then $\angle r$ is also zero. This proves that why light rays do not deviate when they travel normally from one medium to another.

- If the first medium is air, then this refractive index is known as the absolute refractive index of the second medium. Actually refractive index of a medium is its property to decrease the velocity of light. Hence it is also expressed b
- $n_2 = \dfrac{\text{velocity of light in free space}}{\text{velocity of light in median}} = \dfrac{c}{v}$ and its value is always more than 1. Greater the value of refractive index means the second medium has more bending power. Snell's law can be derived from this equation.
- If light ray enters from one medium to another medium in such a way that bending of light is away from normal then second medium is optically rarer than the first medium. If bending of light is towards normal then second medium is optically denser than the first medium.

Principle of Reversibility:

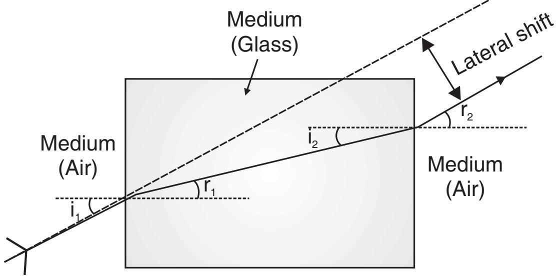
- According to the principle of reversibility the path of light is reversible even if it is going through several mediums. It means light follows exactly the same path when its direction is reversed.
- Applying this rule we may find that if light travels through several medium say medium 1 to medium 2 and then to medium 3 then

$$n_{21} \times n_{32} \times n_{13} = 1$$

- Though refraction rules are universal but direction of emergent ray depends upon the shape of the medium or in other words shape and angle between incident and emergent interfaces (refracting surfaces).

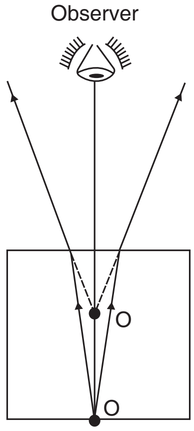
Refraction through glass slab

- In a glass slab refracting surfaces are planes and parallel to each other.
- Emergent ray is parallel to the incident ray but it does suffer lateral displacement.



- The apparent depth of the object is always less than actual depth when looking through glass or water .

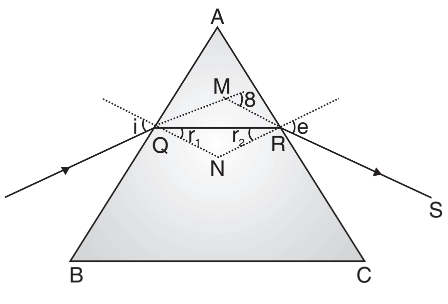
$$\text{Rise of image} = \text{Real depth}\left(1-\frac{1}{n_{21}}\right)$$

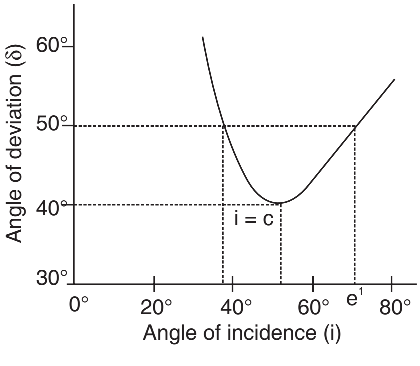


- Following phenomenon occur due to the refraction of light.
  - Bottom of surface of water pool seems to be raised.
  - The letter appears to be raised when we seeing it through a glass slab
  - Objects look bigger than its actual size and raised when we dip it into liquid.
  - Twinkling of stars
  - Delayed sunset and early sunrise

Refraction through Prism

- In prism, refracting surfaces are planes but inclined to each other.
- Refracting ray always bends towards the base.
- Angle of deviation , $\delta = ((i - r_1) + (e - r_2)$
- **Angle of minimum deviation :** When incident angle is increased gradually the angle of deviation initially decreases, and after obtaining a minimum value, it starts increasing again. This minimum deviation is called angle of minimum deviation δ_m.





- At minimum deviation stage it is observed that angle of $i_1 = i_2 \, (= i)$ and $r_1 = r_2 \, (= r)$, then

$$r = \frac{A}{2}$$

$$i = \frac{\delta_m + A}{2}$$

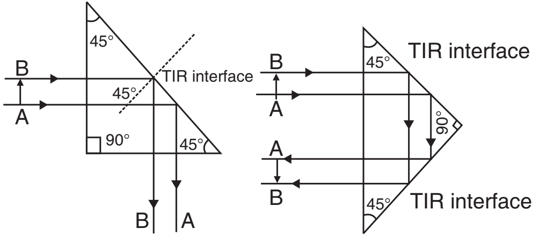
$$n_{21} = \frac{n_2}{n_1} = \frac{\sin[(A + \delta_m)/2]}{\sin[A/2]}$$

As angle of prism and deviation can be found experimentally, this equation is used to determine the refractive index of the material of prism.

- ➢ For thin prism, $\delta_m = (n_{21} - 1)A$. This equation implies that thin prisms do not deviate light much.
- ➢ When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the internal reflection.
  - Critical angle is that value of incident angle for which angle of refraction is 90°. The refracted ray just brushes the surface. The critical angle for water-air, glass-air and diamond-air are 45°, 42° and 24° respectively.

$$n_{12} = \frac{1}{\sin C} \qquad \text{where, } C \text{ is critical angle.}$$

  - If the incidence angle is more than the critical angle, refraction is not possible and incident ray reflects back in rarer medium. This is known as total internal reflection.
- ➢ Hence, conditions for Total internal reflection are
  - Incident ray is in denser medium.
  - Angle of incidence should be larger than critical angle.
- ➢ Natural phenomenon based upon total internal reflection
  - **Mirage :** On hot summer days, light from tall objects successively bends away from the normal due to gradual air density decrease towards the earth. This results total internal reflection and formation of inverted images of distant tall objects. It causes an optical illusion to the observer. This phenomenon is called mirage.
  - **Brilliance of diamond :** Refractive index of diamond is very high ($n \approx 2.42$). Their brilliance is mainly due to the total internal reflection of light inside it.
- ➢ **Application of total internal reflection :**
  - In optical fibers for optical communication.
  - **Prism :** Prisms designed to bend light by 90° or by 180º make use of total internal reflection. Such a prism is also used to invert images without changing their size.



TIR - total internal reflection

- ➢ **Refraction at spherical surface :** If the rays are incident from a medium of refractive index n_1, to another medium of refractive index n_2, the formula comes out to be

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

R = radius of curvature of spherical surface and object is placed at rarer medium.
u = object distance from spherical surface
v = image distance from spherical surface

- ➢ **Lens :** A lens is a piece of transparent glass which is bounded by two surfaces out of which at least one surface is spherical .

  There are two types of lenses
  - **Convex lens :** A convex lens is one which is thinner at sides and thick at centre.
  - **Concave lens :** A concave lens is one which is thicker at sides and thin at centre.
- ➢ **Relation between object distance, image distance with focal length of lens :**

  The relation can be expressed as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

- **Magnification by lens :**

$$m = \frac{\text{height of the image } (h')}{\text{height of the object } (h)} = \frac{v}{u}$$

- **Power of a lens :**
  The power of a lens is defined as the reciprocal of its focal length. It is represented by the letter P. The power P of a lens of focal length *f* is given by

$$P = \frac{1}{f}$$

The SI unit of power is diopter when focal length is in metre. It is denoted by D. Hence one diopter is a power of lens whose focal length is 1 metre.

  - When two or more lenses are combined then the power of combined lens is sum of individual power of lenses.

$$P = P_1 + P_2 + ...$$

- **Lens maker's Formula :**

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \left(n_{21} = \frac{n_2}{n_1}\right)$$

Power of a lens, $P = \frac{1}{f(m)}$.

So, the above formula is used to make lenses of required power. Hence this formula is known as lens maker's formula.

  - **Image formation in convex lens for different positions of object**

	Position of the image	Relative size of the image	Nature of the image
At infinity	at focus F_2	Highly diminished point sized	Real and inverted
Beyond $2F_1$	Between F_2 and $2F_2$	Diminished	Real and inverted
At $2F_1$	at $2F_2$	Same sized	Real and inverted
Between $2F_1$ and $2F_2$	Beyond $2F_2$	Enlarged	Real and inverted
At Focus F_1	At infinity	Infinitely enlarged	Real and inverted
Between focus F_1 and optical	On the same side of the lens as object	Enlarged	Virtual and Erect

  - **Image formation in concave lens for different positions of object**

Position of the object	Position of the image	Relative size of the image	Nature of the image
At infinity	At focus F_1	Highly diminished point sized	Virtual and Erect
Between infinity and the optical center O_1 of the lens	Between focus F_1 and optical center O	Diminished	Virtual and Erect

- **Dispersion of white light through Prism :** Splitting of white colour into its constituent of seven colours is known as dispersion of light. This is due to different colours having different deviations.
  - The seven colours are violet, indigo, blue, green, yellow, orange and red. The acronym of this colour band is **VIBGYOR**.
  - Different colours of light have different wavelengths and different speed in medium. This is the cause of dispersion.
  - In vacuum, the speed of light is independent of wavelength. Thus, vacuum (or air approximately) is a non-dispersive medium in which all colours travel with the same speed. This also follows from the fact that sunlight reaches us in the form of white light (combination of all colours) and not as its components. On the other hand, glass is a dispersive medium.
  - **Angular dispersion through thin prism** = $\delta_v - \delta_R = (n_v - n_R) A$. The relation shows that it depends upon the angle of prism *A*.
  - **Power of dispersion** $\omega = \dfrac{\delta_v - \delta_R}{\delta_y} = \dfrac{n_v - n_R}{n_y}$ is independent of A. It is property of dispersive material.
  - **Recombination of white light :** If we place an inverted identical prism after the first prism, all components colours of light recombine and again became a beam of white light.
- **Phenomenon related to dispersion of light :**
  - **Formation of rainbow :** Rainbow is the natural phenomenon of dispersion of light. After a rain shower when sky becomes clear and sunny, we may observe a rainbow in a direction opposite to the direction of sun when

sun is at our backside. It is caused due to the combined effect of refraction, total internal reflection and dispersion of sunlight by the raindrops suspended in the air.

- In primary rainbow, there is only single total internal reflection before different colours reach observer's eye. In this rainbow, observer see red colour at top and violet at bottom.
- In secondary rainbow, there are two total internal reflections before different colours reach observer's eye. In this rainbow, observer see violet colour at top and red at bottom.
- Secondary rainbow is higher (50° – 53°) on sky than the primary rainbow (40° – 42°).
- Intensity of secondary rainbow is lower than the primary rainbow.

➢ **Scattering of light :** When light deviates randomly from its path due to its interaction with small particles, it is known as scattering of light.

➢ **Tyndall Effect :** The Tyndall effect is the scattering of light as a beam of light passes through a colloid. The individual suspension particles scatter and reflect light, making the beam visible.

➢ The colour of the scattered light depends on the size of the scattering particles.

➢ For $a << \lambda$, where, a is the size of scattering particle, one has Rayleigh scattering which is proportional to $\frac{1}{\lambda^4}$. For $a >> \lambda$, *i.e.,* large scattering objects (for example, raindrops, large dust particles) ; all wavelengths are scattered nearly equally.

➢ **Phenomenon related to scattering of light :**

- Colour of the clear sky is blue.
- Reddening of the Sun at sunrise and sunset.
- White appearance of Clouds.

Know the Terms

➢ The mid-point or the centre of the lens is known as **optical centre** of the lens. It is represented by *O*. A ray of light through the optical centre of a lens passes without any deviation.

➢ The centre of the hollow sphere from which the surfaces of lens are known as centre of curvature. It is represented by *C*.

➢ An imaginary straight line which joins two **centre of curvature** of the lens is known as principal axis. It also passes through optical centre.

➢ **Principal focus :** Incident ray parallel to the principal axis, after refraction from lens either converge to a point (in case of convex lens) after the lens or appear to diverge from a point (in case of concave lens) before the lens. This point is called the principal focus of the lens.

Know the Formulae

➢ **Snell's law of refraction**

➢ $\frac{\sin i}{\sin r}$ = constant (n_{21})

➢ $n_2 = \frac{c}{v}$

➢ $n_{21} \times n_{32} \times n_{13} = 1$

➢ Rise of image = Real depth $\left(1 - \frac{1}{n_{21}}\right)$ (When object is in denser medium and observer is in rarer medium.

➢ Deviation through prism $\delta = (i - r_1) + (e - r_2)$

➢ For thin prism, $\delta_m = (n_{21} - 1)A$

➢ Relation between refractive index, angle of prism and minimum deviation

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin i}{\sin\left(\frac{A}{2}\right)}$$

where

$$i = \frac{\delta_m + A}{2}$$

➢ Condition for Total internal reflection are

- Incident ray is in denser medium.
- Angle of incident should be larger than critical angle.

➢ $n_{21} = \frac{1}{\sin C}$

➢ $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (lens formula)

$$m = \frac{h'}{h} = \frac{v}{u}$$

- Power of lens P = $\frac{1}{f}$
- When two or more lenses are combined then the power of combined lens is sum of individual power of lenses.
  - $P = P_1 + P_2 + ...$
  - Lens maker Formula

$$\frac{1}{r} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \left(n_{21} = \frac{n_2}{n_1}\right)$$

- Angular dispersion through thin prism = $\delta_v - \delta_r = (n_v - n_r)\, A$.
- Power of dispersion $\omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{n_v - n_r}{n_y}$ is independent of A.

TOPIC-3
Optical Instruments

Quick Review

Optical instruments

- Based upon phenomenon of reflecting and refracting properties of mirrors, lenses and prisms, a number of optical devices and instruments have been designed.
- **Microscope** is an optical instrument which help us to see and study micro objects or organism. It forms magnified image of the object.
- **Telescope** is an optical instrument which help us to see and study far off objects magnified and resolved (with clarity).
- We generally set these instruments at two different image vision positions
  - **Image at least distance of distinct vision :** This is the least distance from eye where we able to see objects distinctly. For normal human eye this distance is 25 cm from our eye.
  - **Image at relaxed vision :** This is the distance from eye where we able to see objects distinctly in relax vision (no strain to eye) . For normal human eye this distance is infinity from our eye.
  - Magnification at distinct vision is always greater than magnification at relaxed vision.

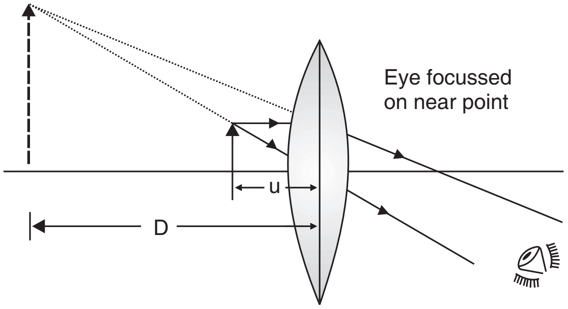
Simple Microscope : Convex lens behaves as simple microscope.

The magnifying power of the simple microscope

For least distance of distinct vision

(i)
$$m = 1 + \frac{D}{f}$$

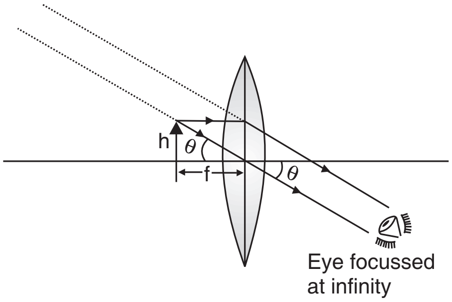
where, D is the least distance of distinct vision of the eye. And f is focal length of the lens.



(ii) For relaxed eye

$$m = \frac{D}{f}$$

from above formulae, it is clear that for larger magnifying power, the focal length of the convex lens should be small.



Please note that angular magnification by optical instruments are linear magnification by lenses only. It means magnification of an instrument means how many times it enlarge the image of object. So this is just as

$$m = \frac{h'}{h}$$

where, h is size of object (in one dimension) and h' is the size of image.

Compound Microscope: For much large magnification, compound microscope is used. It is a combination of two convex lens hence the magnification of each lens is compounded.

- These two lenses are placed co-axially and the distance between them is adjustable.
- The lens towards the object is called objective and that towards the eye is called eyepiece.
- The final image formed by the compound microscope is magnified and inverted.
- Total magnification by compound lens

$$m = m_0 \times m_e$$

where, m_0 is magnification by objective lens and m_e is magnification by eyepiece.

- **For least distance of distinct vision**

Magnification by objective lens is

$$m_0 = \frac{L}{f_0}$$

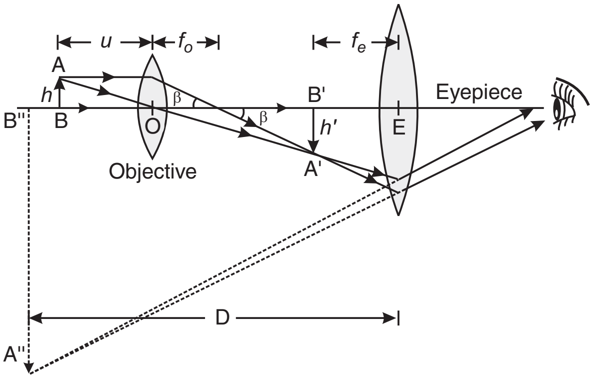
where, L is the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_e). It is called the tube length of the compound microscope.

Eyepiece lens will act as simple microscope.

Magnification by eyepiece lens is

$$m_e = 1 + \frac{D}{f_e}$$

Hence, Magnification by compound lens $= \frac{L}{f_0}\left(I + \frac{D}{f_e}\right)$



➢ **For Relaxed eye (normal adjustment)**

For relaxed eye the magnification by objective lens remain same, the magnification by eyepiece will be $+\frac{D}{f_e}$

Hence, the total magnification of compound microscope in relaxed eye condition is $m = \frac{L}{f_0} \times \frac{D}{f_e}$

➢ **Properties of Compound Microscope :**

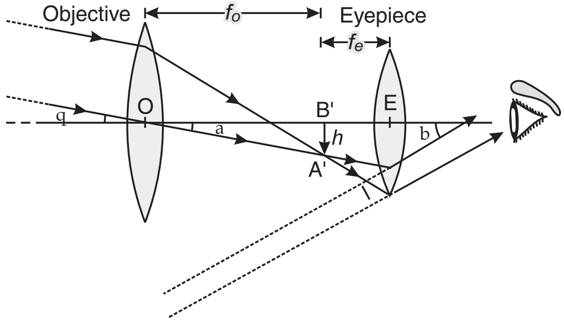
- For large magnification of a compound microscope, both f_0 and f_e should be small.
- If the length of the tube microscope increases, then its magnifying power increases.
- Generally f_0 is much smaller. So that objective is placed very near to principal focus.

  - The aperture of the eyepiece is generally small so that whole of the light may enter the eye.
  - The aperture of the objective is also small so that the field of view may be restricted.
- **Magnification by Telescope :**
  - Telescope is an instrument to magnify and resolve far off objects.
  - Far off objects make much smaller angle at our eye. Telescope make that angle bigger without much intensity loss.
  - To maximise the intensity aperture size of objective lens is quite large. It will focus a bright point size image at its focal plane.
  - Now with eyepiece, we will form this point size image to final inverted magnified image. This type of telescope is known as astronomical telescope.
- **For least distance of distinct vision**

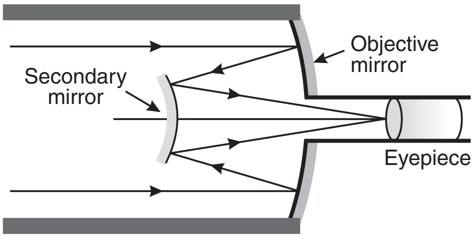
$$m = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

- **For relaxed eye (normal adjustment)**

$$m = -\frac{f_o}{f_e}$$



- **Properties of astronomical telescope :**
  - For larger magnifying power, f_o should be large and f_e should be small.
  - The length of the tube of an astronomical telescope is $L = f_o + f_e$ for relaxed vision adjustment.
  - When the length of the tube of the telescope increases, f_o increases and hence the magnifying power also increases. Note that f_e is always small.
- **Limitations of refractive telescope :**
  - Large objective lens makes the telescope very heavy. So it is difficult to handle it by hand.
  - It has spherical and chromatic aberrations.
- **Modern Telescope (Reflective Telescope)**
  - Reflecting telescope consists of a concave mirror of large radius of curvature in place of objective lens
  - A secondary convex mirror is used to focus the incident light, which now passes through a hole in the objective primary mirror .
  - The magnifying power of the reflecting telescope is $m = \frac{f_o}{f_e}$



- **Advantages of reflective telescope :**
  - Very sharp point image by objective mirror removes spherical aberrations.
  - As it is very light so large aperture of parabolic mirror can be used for desired magnification.
  - This is based on the principle of reflection hence there will be no chromatic aberrations.

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Chapter - 10 : Wave Optics

TOPIC-1

Wave theory and Huygens Principle

Quick Review

- Newton's supported 'Descartes corpuscular theory' of light and further developed it.
- According to the corpuscular theory "sources of light emit large number of tiny massless particles known as corpuscles in a medium surrounding the source. They are perfectly elastic, rigid and have high speed.
- This theory could explain reflection and refraction of light but could not explain many other optical phenomenon like interference and diffraction of light. It was also unable to explain partial reflection and refraction through a transparent surface.
- Huygens proposed wave theory of light. According to his theory, light travels in the form of longitudinal waves with uniform speed in a homogenous medium. Different wavelengths of light represent different colours of light.
- As longitudinal mechanical waves need medium to travel, he assumed a hypothetical medium known as 'ether'. He also proved that speed of light is slower in optically denser medium.
- Initially, Huygens wave theory of light couldn't get much success. Its main point of rejection was, it was considered as longitudinal wave which need medium, but experimentally found that it could also travel in vacuum and there is no medium like ether.
- But later Maxwell theory of electromagnetic waves and Young's famous double slit experiment firmly established this theory. Maxwell explained light is an electromagnetic wave which does not need medium and its speed in vacuum is 3×10^8 m/s. Till date phenomenon of optical interference, diffraction and polarization can be explained with wave nature of light.
- It had some points of failure also. It could not explain photoelectric effect and Compton effect.
- With polarization phenomenon, it is established that light is not a longitudinal wave but it is a transverse wave.
- Huygens principle brings concept of formation of new wave-fronts and its propagation in forward direction.
- Wavefront is locus of all points in which light waves are in same phase. Propagation of wave energy is perpendicular to the wavefront.

Huygens Principle :

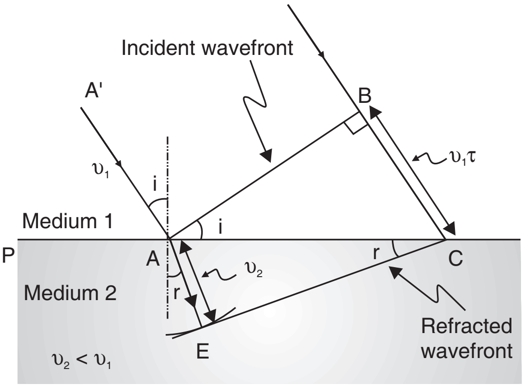
- Every point of a wavefront becomes secondary source of light.
- These secondary sources give their own light waves. Within small time they produce their own wave called secondary wavelets. These secondary waves have same speed and wavelengths as waves by primary sources.
- At any instant a common tangential surface on all these wavelets give new wavefronts in forward direction.
- **Shapes of wavefronts**

Source	Wavefronts
Point source	Spherical wavefront
Line source	Cylindrical wavefront
Plane source	Plane wavefront
Point source is very far away	Plane wavefront

- Concave lens converts plane wavefront to convex wavefront and Convex lens convert plane wavefront to concave wavefront
- Refraction of a plane wave by Huygens Principle :
- Snells law can be proved by Huygens principle.

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant}$$

- It is also proved that the velocity of light in denser medium is less than velocity of light in rarer medium.



AB = incident wavefront

EC = refracted wavefront

$\angle i$ = angle between incident wavefront *AB* and interface *PP*′

$\angle r$ = angle between refracted wavefront *EC* and interface *PP*′

If medium 2 is optically denser than medium 2 and τ is the time in which disturbance from *B* reaches at *C*. This is the same time in which disturbance from *A* reaches at *E* where distance *AE* < *BC*.

$$\Delta AEC \sim \Delta ABC$$

$$\sin i = \frac{BC}{AC}$$

$$\sin r = \frac{AE}{AC}$$

$$\frac{\sin i}{\sin r} = \frac{BC}{AE}$$

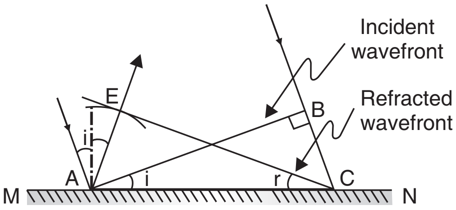
BC = Distance travelled by disturbance at *B* in time τ in medium 1

AE = Distance travelled by disturbance at *A* in time τ in medium 2

$$\frac{\sin i}{\sin r} = \frac{v_1\tau}{v_2\tau}$$

Hence $$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant}$$

- This is law of refraction (snell's law).
- **Reflection of a plane wave by Huygens Principle**



AB = incident wavefront

EC = reflected wavefront

$\angle i$ = angle between incident wavefront *AB* with the interface *AC*

$\angle r$= angle between reflected wavefront *EC* with the interface *AC*

If disturbance at A is reflected back from the interface *AC* then disturbance at *B* and disturbance at *A* both travel in medium 1 and they will have travelled equal distance in medium 1 in time τ. Where τ is the time in which disturbance from *B* reaches at *C*.

Now *AE* = *BC* (distance travelled in same medium in same time)

$$\Delta AEC \cong \Delta ABC$$

$$\angle i = \angle r$$

This is law of reflection.

TOPIC-2

Superposition of light waves (Interference and Diffraction)

Quick Review

- According to superposition principle, "At a particular point in the medium, the resultant displacement produced by a number of waves is the vector sum of the displacements produced by each of the waves".
- It means if individual displacement produced at a point by two coherent waves at any instant is given by

$$y_1 = a\cos\omega t \text{ and } y_2 = a\cos\omega t.$$

Then resultant displacement at that point will be

$$y = y_1 + y_2 = 2a\cos\omega t.$$

Hence the total intensity at that point will be :

$$I = 4\,I_0$$

where, $I_0 \quad a^2$; maximum intensity due to one wave.

Interference

- **Constructive superposition :** If two waves are propagating such that crest and trough of both waves would reaching at a point in the same instant then we say there is constructive superposition of two waves at that point and the resultant amplitude of the wave is the sum of individual amplitudes. (We can generalize this to superposition of more than two waves) $a = a_1 + a_2$
- Constructive interference : Waves would be coherent in nature. Coherent wave means they should have equal frequency and constant phase difference with each other at any time interval t.
- **Destructive superposition :** If two waves are propagating such that crest of one wave and trough of other wave reaching at a point in different instant then we say there is destructive superposition of two waves at that point. The resultant amplitude of the wave is the difference of individual amplitudes. (We can generalize this to superposition of more than two waves)
- Any two independent sources can never be coherent. We may create two coherent sources by deriving them from one source.

Condition for constructive interference

- Waves would be coherent in nature. Coherent wave means they should have equal frequency and constant phase difference(0, 2π, --- ($2n\,\pi$) with each other at any time interval t.
  Path difference between waves at this phase difference = 0 , λ, -----($2n - 1$)λ

$$a_r = a_1 + a_2$$

if $a_1 = a_2$

then $a_r = 2a$

$\because$ $I \propto a^2$

$$I_r = 4a^2$$

Condition for destructive interference

- Waves would be coherent in nature. The phase diff. of the waves should be odd multiples of π, *i.e.*, 0, π, ... $(2n-1)\pi$
- Path difference between waves at this phase difference = 0, $\frac{\lambda}{2}$, ...$(2n-1)\frac{\lambda}{2}$

$$a_r = a_1 - a_2$$

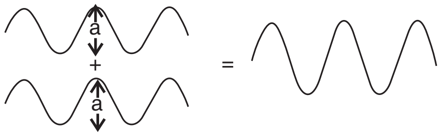
if $a_1 = a_2$

then $a_r = 0$

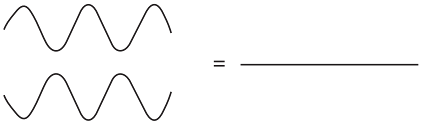
$\because$ $I \propto a^2$

$$I_r = 0$$

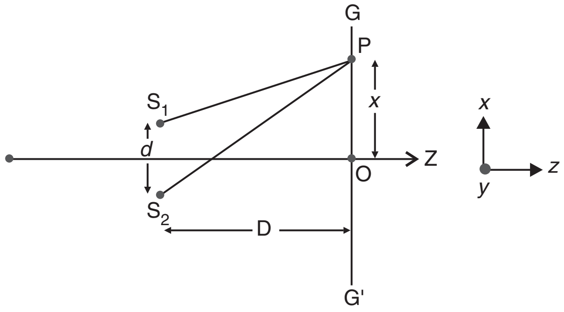
Constructive Interference



Destructive Interference



Young's Experiment :



- At "O" we would get central maxima. Here path difference $(S_2P - S_1P) = 0$
- At "P", which is at "x" height from "O" path difference $(S_2P - S_1P) = \frac{dx}{D}$
- **Condition for P is a bright spot :**

$$\frac{xd}{D} = 0, \lambda, 2\lambda .... n\lambda$$

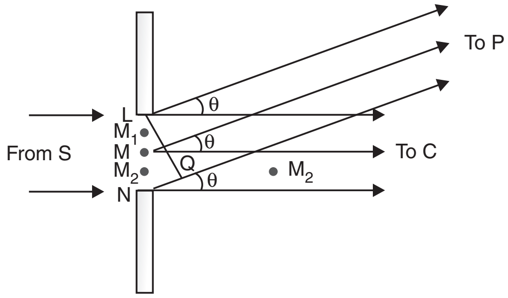
$$x_{nth\ bright} = \frac{nD}{d}\lambda$$

where n is number of bright fringe after central fringe.

$$\frac{xd}{D} = 0, \frac{3\lambda}{2} --- (2n+1)\frac{\lambda}{2}$$

$$x_{nth\ dark} = \frac{(2n-1)D}{2d}\lambda$$

- Width of the bright fringe $(W_B) = x_{nD} - x_{(n-1)D} = \frac{D\lambda}{d}$
- Width of the dark fringe $(W_O) = x_{nB} - x_{(n-1)B} = \frac{D\lambda}{d}$
- Width of the central fringe $(W_C) = \frac{D\lambda}{d}$
- Hence $W_B = W_D = W_C$
- **Diffraction :** It is defined as the bending of light around the corners of an obstacle or aperture into the region where we should expect shadow of the obstacle.



If width of the opening = a

Path difference between the ray at A and the ray at N = θ where θ is the angle of elevation of point P from principal axis.

Path difference between ray from L and ray from N = $LQ = a \sin\theta$

$$a \sin\theta = \lambda \quad \because \text{ for first maxima } \sin\theta <<< 1 \qquad (\therefore \sin\theta \cong \theta)$$

$$\theta = \frac{\lambda}{a}$$

It is observed that when path difference = λ, 2λ ...... $(2n - 1)\lambda$ then P is a dark point.

When = $a\sin\theta = \frac{3\lambda}{2}, ...(2n+1)\frac{\lambda}{2}$

Elevation angle for first bright fringe $\theta_{1B} = \frac{3\lambda}{2a}$

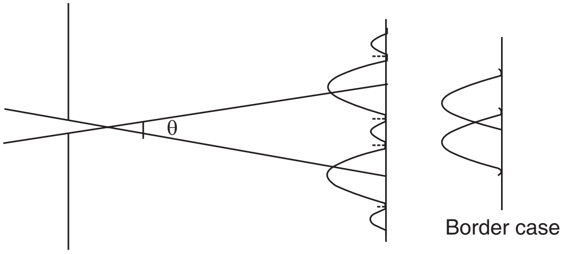
- Height of first bright fringe $x_{1B} = \frac{3\lambda D}{2a}$ where, θ D is the distance between screen and opening a.
- Elevation angle for first dark fringe $\theta_{1D} = \frac{\lambda}{a}$
- Width of the bright fringe = $\frac{D\lambda}{a}$
- Width of the dark fringe = $\frac{D\lambda}{a}$
- Width of the central fringe = $\frac{2D\lambda}{a}$
- There is no gain or loss of energy in interference or diffraction, which is consistent with the principle of conservation of energy. Energy only redistributes in these phenomena.

TOPIC-3

Resolving Power of Optical instruments and Polarisation of Light

Quick Review

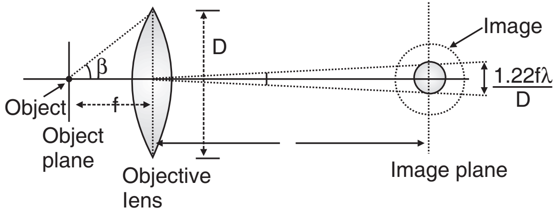
- **Resolving Power of Optical Instruments :** The ability of an optical instrument to separate small or closely adjacent images is resolving.
- **Border line resolving** is the required minimum angle θ between two objects so that they may be resolved.



- **Relaigh criteria for border line resolving :**
  If opening is circular (which is case of most optical instruments), Relaigh criteria of border line resolving is

$$\Delta\theta = \frac{1.22\lambda}{a} = \frac{D}{d}$$

- **Resolving Power of Microscope :**



$$d_{min} = \frac{1.22\lambda}{2\sin\beta}$$

And $P \propto \frac{1}{d}$ (the minimum distance it may resolve, the more powerful is the microscope). Hence we may reduce the value of λ by filled the liquid of refractive index n around the objective lens. In this case the medium between the object and the objective lens is not air but a medium of refractive index n.

$$d_{min} = \frac{1.22\lambda}{2n\sin\beta}$$

- **Resolving Power of Telescope :**

$$\Delta\theta_{\text{min}} = \frac{1.22\lambda}{a}.$$

Hence to differentiate minimum angular resolution, aperture *'a'* of the objective lens should be as larger as possible.

$$P = \frac{a}{1.22\lambda}$$

- **Fresnel's distance limit of ray optics :**

$$Z = \frac{a^2}{\lambda}$$

where, a is the size of aperture.

Doppler's effect :

When there is relative motion between source of light and observer, the frequency as apparent to observer is different and there is change, according to the following relation.

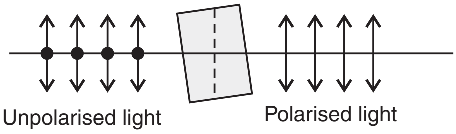
$$\frac{\Delta v}{v} = \frac{-v}{c}$$

where, v = original frequency
Δv = change in frequency
v = relative velocity. It is considered to be positive when source is moving away from the observer.

If there is redshift in doppler effect then source is moving away and if there is blue shift in doppler effect then source is moving towards observer.

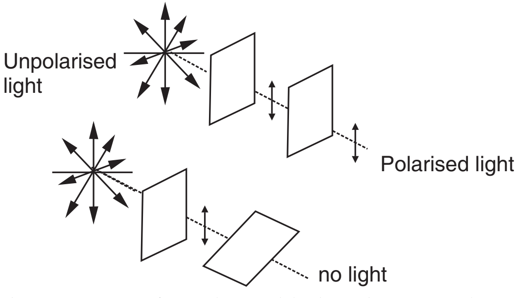
Polarisation :

- In an unpolarized wave, the displacement will be randomly changing with time though it will always be perpendicular to the direction of propagation.
- Hence in unpolarised light wave, vibrations of electric fields are in more than one direction. Polarized light waves are light waves in which the vibrations occur in a single plane.
- The process of transforming unpolarized light into polarized light is known as polarisation.
- Light is unpolarized by nature but it may be converted into fully or partial polarized light
- The light having oscillation only in one plane is called polarised or plane polarised.
- The plane perpendicular to the plane of oscillation is called plane of polarisation.
- Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.
- Polaroids are thin films of ultramicroscopic, crystals of quinine iodosulphate with their optical axes parallel to each other.
- Polaroids allow the light oscillations parallel to the transmission axis to pass through them.
- If an unpolarised light is incident on a polaroid, the transmitted light is plane polarised as shown below :



Here, the vertical oscillations are transmitted because the transmission axis is also vertical. The horizontal oscillations are not transmitted. That is why, on the right hand side there are no dots at the intersection of lines.

- When polaroids are used to convert unpolarised light into polarised they are known as polarisers.
- When polaroids are used to detect polarised light, they are known as analyser.
- If the transmission axes of the polariser and analyser are parallel, then whole of the polarised light passes through the analyser.
- If the transmission axis of the analyser is perpendicular to that of polariser, then no light passes through the analyser. Such polariser and analyser are said to be crossed.



- **Malus's law :** It states about the intensity of a polarised light when it is altered to pass through a polariser at an angle θ. According to this law " If I_0 be the intensity of the polarised light incident on the analyser and θ be the angle between the transmission axes of the polariser and analyser, then the intensity of the light transmitted through the analyser is given by $I = I_0\cos^2\theta$."

$$\text{Percentage of polarisation } = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \times 100$$

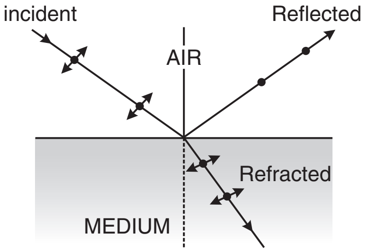
- **Polarisation by reflection :** When unpolarised light is incident on the boundary between two transparent media such that reflected wave is perpendicular to the refracted wave. Then the reflected wave is a totally polarised wave. The angle of incidence in this case is called Brewster's angle and is denoted by i_B. If refractive index of second medium is μ then

$$\mu = \frac{\sin i}{\sin r}$$

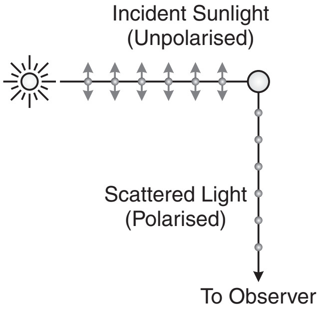
$$\mu = \frac{\sin i_\beta}{\sin(90 - i_\beta)}$$

$$\mu = \frac{\sin i_\beta}{\cos i_\beta} = \tan i_\beta$$

This is known as **Brewster's law.**



- **Polarisation by scattering :** When scattered light coming to the observer such that source is at 90°, then scattered light coming to observer is polarised.



- **Applications of Polarisation :**
- Polaroids are used to reduce the intensity in window pane, sunglasses etc.
- Polaroids are also used in photographic cameras and 3D movie cameras.
- Used in making holograms.
- Polarisation proves that light is transverse wave.
- Chemical analysis of molecules.

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UNIT - VII : Dual Nature of Radiation and Matter

Chapter - 11 : Dual Nature of Radiation and Matter

TOPIC-1

Photoelectric Effect

Quick Review

- In attempt towards unification of study of physics, it was established in 19th century that everything in nature can be classified into either matter or radiation.
- Several fruitful experiments were carried out independently on matter and radiations during that time. In 1897, Maxwell established electromagnetic theory which unified all radiations like light and heat.Maxwell established the wave theory of light. X-ray radiation was also discovered during that time in 1895.

- Simultaneously in study of matter, a milestone discovery of electron was done by J.J. Thomson in 1897. It established that atoms of different matters constitute same particles and one of them is electron.
- **Electron Emission :** Electron has two types of motion orbital or zig-zag in free state depending upon its energy. Free electrons have higher energy than orbital electrons.
  - Free electrons in metals cannot come out from the surface due to force by positive ions present in metals. Electron can come out of the metal surface only if it has got sufficient energy to overcome this attractive pull.
  - **Work Function of a metal :** Work function is the minimum amount of work done (energy given) to an electron so that it can escape the metal surface. Work function is different for different metals. It is measured in electron volt (eV).
  - One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt.

$$V = \frac{W}{e} \quad (q = e\text{, for an electron})$$

When $V = 1$; $W = 1$ eV; putting these values in equation

Hence $1\text{ eV} = 1.6\times 10^{-19}$ J

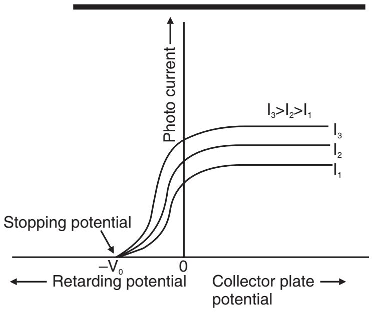
- There are three types of electron emissions

- **Thermionic emission :** When electron emission occurs by heating the metal, then it is known as thermionic emission. Emitted electrons are called thermionic electrons.
- **Field emission :** When electron emission occurs by applying strong electric field, then it is known as field emission and emitted electrons are called field electrons.
- **Photoelectric effect :** When electron emission is occurred by illumination of metal by light of suitable frequency, then it is known as photoelectric emission. Here, emitted electrons are called photo electrons.
  - When light falls on the metal surface, free electrons absorbs energy from light and if this energy is more than the work function of metal, the electron escapes from the surface. This phenomenon is known as photoelectric emission. This was first observed by Hertz.
- **Hallwachs' and Lenard's detailed study of Photoelectric effect:**
  - In 1888, Lenard observed that when ultraviolet light falls on zinc metal, metal becomes positively charged. With the discovery of electrons it was established that this is due to emission of electrons. The current produced by these photoelectric electrons is called photoelectric current.
  - When the frequency of the incident light is smaller than a certain minimum value, called the threshold frequency no emission of electrons take place.
- **Hertz and Lenard's experiment :** This experiment led the formation of quantum theory of light as wave theory could not explain photoelectric effect.
  - Experiment was carried to study the following two properties of light :
  - **Intensity of light :** Power of light is directly proportional to the intensity of light. A higher power bulb (say 100 watt) has more intensity than the lower power bulb (say 50 watt).
  - **Frequency of light :** Colour of light is due to its characteristic property of frequency.

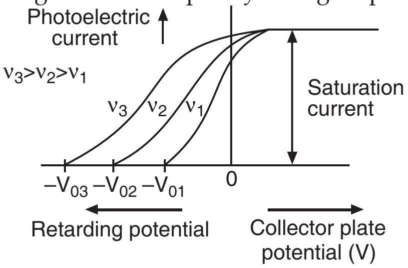
    $\nu = \frac{c}{\lambda}$ where, υ is frequency of light.

- **Experiment outcome :** It showed that intensity of light has linear relationship with photoelectric current at potential higher that stopping potential.
- **Effect of Potential on photoelectric current.**

  For a given frequency of the incident radiation, the stopping potential is independent of its intensity.



- **Effect of frequency of incident radiation on stopping potential :**
  - It was observed that photoemission current starts only at certain minimum frequency of light known as **threshold frequency** of that metal. Below this frequency, photoemission does not take place whatever be the intensity of light of that frequency falling on photosensitive plate.



- Wave theory was inadequate or failed to explain the photoelectric effect due to the following reasons :
  - According to wave theory, higher amplitude is higher energy but experiment show that even larger amplitude (high intensity) of light below threshold frequency could not give photo electric effect.
  - According to wave theory, same intensity of different light colour should have same energy but experiment shows that energy depends upon frequency not amplitude.
  - According to wave theory, wavefront should take some time to give energy to electron but experimentally found that ejection of electron is instantaneous.
- In 1900, Max Planck stated that electromagnetic energy could be emitted only in quantized form.

$$E = hv$$

where, h is Planck's constant.

- Based upon this postulate Einstein established quantum theory of radiation and was able to explain photoelectric phenomenon by this theory. It states that light energy packets are photons (Particle nature of light).
- In photoelectric effect, an electron absorbs a quantum of energy ($E = h\nu$) of radiation. If this absorbed quantum of energy exceeds the minimum energy needed for the electron to escape from the metal surface (work function ϕ_0), the electron is emitted with maximum kinetic energy.

$$K.E. = h\nu - \phi_0$$

where, ϕ_0 is the work function of the metal.

- At stopping potential, kinetic energy of ejected electron is zero. Below this potential electrons can not be ejected. Hence, maximum kinetic energy of an electron is calculated by

$$\text{K.E.}_{max} = eV_0$$

where, V_0 is stopping potential.

Work function of metal $\phi_0 = \upsilon_0 h$

where, ν_0 is the cutoff frequency.

- Maximum speed of emitted photoelectrons can be calculated as

$$v_{max} = \sqrt{\frac{2KE_{max}}{m}}$$

- According to quantum theory all photons of specific light frequency have equal energy. Intensity of light only increases the number of photons per unit area not the energy of photons.
- Photons are electrically neutral and are not deflected by electric and magnetic field.
- Photon has energy to propagate hence it has momentum.

  Momentum of photon $p = \dfrac{h\nu}{c}$

- In photon- electron collision, number of electrons or photons are not conserved but energy and momentum are conserved.
- As Interference, diffraction and polarization cannot be explained by quantum theory of light, hence it was said that light has dual nature. When it travels in a medium, it travels as wave, while interacting with other medium it acts like particles (photons).

TOPIC-2

Dual Nature of Matter

Quick Review

- de- Broglie's postulate is based upon the symmetry of nature. If radiation has dual nature then matter should also have dual nature.

- According to his hypothesis moving particles of matter should display wave nature under suitable condition. He named this wave as matter wave. It is a third type of wave. It is different from mechanical wave and electromagnetic wave.
- **Properties of matter wave :** Whenever a particle moves, the matter wave envelops it and controls its motion.
  - de-Broglie proposed that the wave length λ known as de-Broglie wavelength; associated with momentum of particle p is given as

$$\lambda = \frac{h}{p}$$

Hence, de-Broglie's wavelength of particle

$$\lambda = \frac{h}{mv}$$

  - **Calculation of electron wave :**
    In photoelectric equation; kinetic energy of electron at potential V is $K = eV$. Putting this value of kinetic energy in de-Broglie wavelength equation

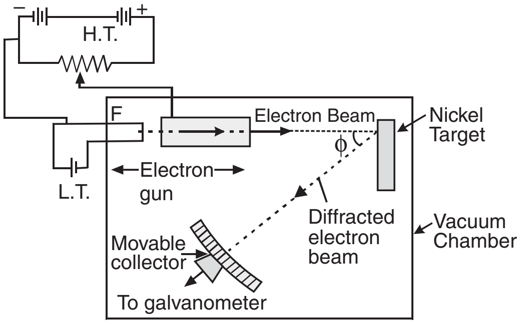
$$\lambda_e = \frac{h}{\sqrt{2meV}}$$

By putting the value of mass of electron, its charge and Plancks' constant $\lambda_e = \frac{1.227}{\sqrt{V}}$ nm. This is theoretical calculation of de-Broglie wavelength of electron. where, V is the magnitude of accelerating potential in volts.

  - From this formula; wavelength of particle is inversely proportion to the mass of particle and its velocity. Hence. heavier particles have shorter wavelengths.
- **Heisenberg Principle of uncertainty :**
  - According to Heisenberg's uncertainty principle, it is not possible to measure both the position and momentum of an electron (or any other particle) at the same time exactly. When any particle is in motion then product of uncertainty of its momentum and uncertainty is given as

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

  - According to Heisenberg's hypothesis, electron is not moving in an orbit but it is moving in a wave packet cloud.
- **Davisson and Germer Experiment :** Davisson and Germer did the experiment to see the diffraction pattern for electron waves and verified for first time the wave nature of electron.



- In their diffraction experimental setup, interatomic space between Nickel is taken as single slit. Scattered electron beams are collected by movable collector and its intensity is plotted in a radial graph.
- In radial graph we take the observations at different angles and plot the graph.
- The experiment was performed by varying the accelerating voltage from 44 V to 68 V.
- A strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54 V at a scattering angle $\theta = 50°$
- The contrast of concentration of scattered electrons proved that electrons are moving in waveform and superposition of their wave appear as contrast of concentration of scattered electrons.
- Calculation of electron wavelength by wave optics diffraction formula and putting experimental values as for 1st dark diffraction angle 50° and interatomic space of nickel $d = 0.91$ A° gives $\lambda = 0.165$ nm.
- Theoretical calculation of electron wavelength by de-Broglie formula $\left(\lambda_e = \frac{1.227}{\sqrt{V}}\text{nm}\right)$ by putting electrical potential value $V = 54$ volts gives electron wavelength as 0.167 nm.
- This excellent agreement between theoretical and experimental value of electron wavelength confirms the wave nature of electrons and the de-Broglie relation.
- The wave properties of electrons have been utilized in the design of electron microscope which have very higher resolution, than the optical microscope.

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UNIT - VIII : Atoms & Nuclei

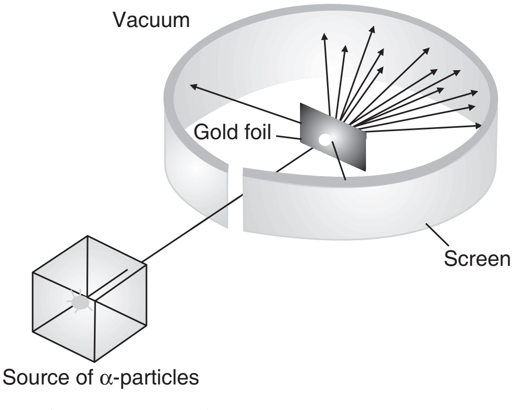
Chapter - 12 : Atoms

Quick Review

- We are surrounded by uncountable things (living or nonliving) : If we breakup matters round us into molecules, we can have millions of different molecules.
- Further breakdown of molecules into atoms we find there are only roughly hundred types of atoms (atom is the identity of an element. 115 types of elements are known to us till date.)
- By the nineteenth century, enough evidence had accumulated in favour of atomic hypothesis of matter. It says that atom is smallest particle and atom is indivisible.
- All atoms radiate different light spectra which shows these atoms are different and may be smallest particle.
- With the discovery of electron by J.J. Thomson, It was evident that atoms have identical sub-particles and different light spectra of different atoms are due to the motion of this sub-particle.
- **Atomic models**
- As atom is neutral so discovery of electron led by J.J. Thomson to establish that it should also have positive charge. Hence he proposed first model of atom – Plum pudding model.
- **Plum-Pudding model :** According to plum pudding model "the positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon."
- But subsequent studies on atom showed result very different to this atomic model.
- **Rutherford atomic model :**
- With the discovery of Avogadro number, calculation of atom size was very big as compared to the sizes of atomic sub-particles.
- This led Rutherford to establish the theoretical second atomic model known as "nuclear model of the atom". It was inspired by planetary position around Sun.
- According to this model " The entire positive charge and most of the mass of the atom is concentrated in a small volume called the nucleus with electrons revolving around the nucleus just as planets revolve around the sun."
- Though it was initially a theoretical model but it was a major step towards the modern atomic model.
- **Geiger** and **Marsden** experimentally proved Rutherford atomic model.

Geiger & Marsden scattering experiment :

- **Experimental setup :**



- Radioactive element $^{214}_{83}Bi$ was taken as α-particles generating source.
- Gold was taken as target metal. Selection of gold was based upon its two important characteristics :
- Highest malleability hence very thin foil is possible to make. Gold foil used was almost transparent.
- Gold is a heavy metal hence it helped in discovery of nucleus.
- Lead bricks absorbed the α-particles which were not in gold foil direction. They worked as collimator.
- Detector was made from ZnS.

Experimental observations :

- When α-particles hit ZnS screen, it absorbs and glow. Hence with intensity we can count the number of α-particles.

- Most of the α-particles passed through roughly in straight line (within 1°) without deviation. This showed that no force acting upon most of α-particles.
- A very small number of α-particles were reflected. (1 out of 8000)

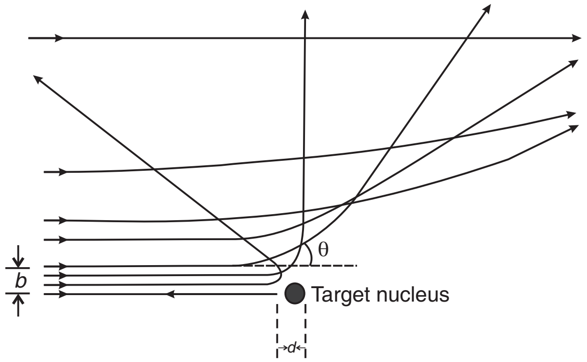
Conclusions :

- Space in the atom is mostly empty (only 0.14% scatters more than 1°)
- Experiment suggest that all positively charged particles are together at one location at centre. It was called nucleus. So, nucleus had all the positive charges and the mass. Therefore, it had capability to reflect heavy positive α-particles.
- Size of nucleus calculated to be about 10^{-14} m. According to kinetic theory, size of one atom is of the order of 10^{-10} m.
- Force between α-particles and gold nucleus

$$F = \frac{1}{4\pi\varepsilon_0}\frac{2eZe}{r^2}$$

Alpha-particle trajectory :

- **Impact parameter :** It is the perpendicular distance between direction of given α-particle and centre of nucleus. It is represented by '*b*'.
- **Distance of closest approach :** It is the distance between centre of nucleus and the α-particle where it stops and returns back. It is represented by '*d*'. This distance give approximation of nucleus size.



Electron Orbits

- We can calculate the energy of an electron and radius of its orbit based upon Rutherford model.
- The electrostatic force of attraction, F_e between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to keep them in their orbits.

$$F_e = F_c$$

For hydrogen atom
$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

Or
$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

Electron has kinetic energy, $K = \frac{1}{2}mv^2$. Putting the value of mv^2 in the above equation

$$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

And
$$v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$$

P.E. of an electron, $U = -\frac{1}{4\pi\varepsilon_0 r}.\frac{e^2}{r}$ (negative sign shows that its due to attraction force)

Total energy,
$$E = K + U$$

$$E = \frac{e^2}{\pi 8\varepsilon_0 r} + \left(-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right)$$

$$= -\frac{e^2}{8\pi\varepsilon_0 r}$$

- Due to this negative energy, electron is bound to nucleus and revolves around it. This energy is known as binding energy of electron.

- From the equation, it is clear that if energy is zero then radius is infinity. Practically it means if we provide this amount of energy to this electron, it gets free.

Atomic Spectra :

- Each element has a characteristic spectrum of radiation, which it emits There are two types of atomic spectra. Emission atomic spectra and absorption atomic spectra.
- **Emission atomic spectra :** Due to excitation of atom usually by electricity, light of particular wavelength is emitted, this type of atomic spectra is known as emission spectra.
- **Absorption Atomic Spectra :** If atoms are excited in presence of white light, it absorbs its emission spectral colour and black line will appear in the same place of that atoms' emission spectra. This type of spectra is known as absorption spectra.

Spectral series :

- Any atom shows range of spectral lines. Hydrogen is the simplest atom and hence simplest spectrum.
- The spacing between lines within certain sets of the hydrogen spectrum decreases in a regular way. Each of these sets is called a spectral series.
- **Balmer Series :** Balmer observed the first hydrogen spectral series in visible range of hydrogen spectrum. It is known as Balmer Series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Longest wavelength = 6566.4 Å
Shortest wavelength = 3648 Å
where, R is Rydberg's constant . The value of R is $1.097 \times 10^7\ m^{-1}$; $n = 3, 4, 5...$

$$\frac{1}{\lambda} = \frac{\nu}{c}$$

Hence

$$\nu = R_c\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

- Other series of spectra for hydrogen were as follows

Lyman Series : $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$; $n = 2, 3, 4, 5, ...$ This is in UV range.

Longest wavelength = 1216 Å
Shortest wavelength = 912 Å

Paschen Series : $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$; $n = 4, 5, 6$

Longest wavelength = 18761.14 Å
Shortest wavelength = 8208 Å

Brackett Series : $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$; $n = 5, 6, ...$

Longest wavelength = 40533.33 Å
Shortest wavelength = 14592 Å

Pfund Series : $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$; $n = 6, 7, 8 ....$

Longest wavelength = 74618.1 Å
Shortest wavelength = 22800 Å

The Lyman series is in the ultraviolet, and the Paschen and Brackett series are in the infrared region.

Limitation of Rutherford model :

- **It could not explain the stability of the atom :** The electron orbiting around the nucleus radiates energy. As a result, the radius of the electron orbit should continuously decrease and ultimately the electron should fall into the nucleus.
- **It could not explain nature of energy spectrum :** According to the Rutherford's model, the electrons can revolve around the nucleus in all possible orbits. Hence the atom should emit radiations of all possible wavelengths. However, in practice, the atoms are found to have line spectrum or discrete spectrum.

Bohr Model & Bohr's Postulate :

- An electron in an atom could revolve in certain stable orbits without radiant energy in certain stable orbits without emission of radiant energy. These orbits are called stationary states of atom.

- Electron revolves around nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$, where, *h* is Planck's constant.
- Hence angular momentum, $L = \frac{nh}{2\pi}$
- An electron may make a transition from one of its specified non-radiating orbit to another of lower energy. When it does so, a photon is radiated having energy equal to energy difference between initial and final state.

$$h\nu = E_i - E_f \qquad \text{where, } \nu \text{ is frequency}$$

Angular momentum $L = mv_n r_n$

According to Bohr's postulate $L = \frac{nh}{2\pi}$

Hence,

$$mv_n r_n = \frac{nh}{2\pi}$$

$$mr_n = \frac{nh}{2\pi v_n}$$

For hydrogen atom

$$v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$$

Combining these two equations, we get

$$v_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{(h/2\pi)}$$

This equation depicts that electron speed in nth obit falls by a *n* factor.

And

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2\frac{4\pi\varepsilon_0}{e^2}$$

For innermost orbit $n = 1$; the value of r_1 is known as Bohr's radius a_0.

$$a_0 = \frac{h^2\varepsilon_0}{\pi me^2}$$

If we put values of all constants, we get $a_0 = 5.29 \times 10^{-11}$ m.

It can also be observed that radii of n^{th} orbit increase by n^2 times.

By putting this value in total energy of an electron and convert the unit in eV, we get

$$E_n = \frac{-13.6}{n^2}\text{eV}$$

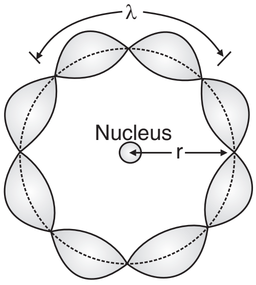
Negative value shows electron is bound to nucleus.

- The explanation of the hydrogen atom spectrum provided by Bohr's model was a brilliant achievement.

De Broglie's Explanation of Bohr's second postulate by quantisation theory :

- According to Bohr's postulate, electron in hydrogen atom can revolve in certain orbit only in which its angular momentum, $L = n\frac{h}{2\pi}$. In these stationary orbits electron does not radiate emission.
- De- Broglie proves this with the help of wave nature of electron while revolving.
- Travelling wave propagates energy but stationary wave does not. In analogy to waves travelling on a string, particle waves too can lead to standing waves under resonant conditions. Resonant condition is

$$l = n\lambda$$



Standing wave when l = n λ

For hydrogen atom, length for the innermost orbit is its perimeter. hence

$$2\pi a_0 = n\lambda \qquad ...(i)$$

According to de-Broglie wavelength of electron, $\lambda = \frac{h}{p}$. so,

Now equation (i) can be written as

$$2\pi r = n\frac{h}{p} \qquad ...(ii)$$

But $p = mv$

Hence, equation (ii) can be reduced as, $2\pi r = n\frac{h}{mv}$

$$mvr = \frac{nh}{2\pi}$$

$$L = \frac{nh}{2\pi}$$

This is Bohr's second postulate.

Limitation of Bohr's atomic model :

- Bohr's model is for hydrogenic atoms. It is not correct for multi-electron model.

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Chapter - 13 : Nuclei

TOPIC-1
Nucleus and Mass Energy Relation

Quick Review

- With Rutherford scattering experiment, it is established that radius of atom is 10^4 times of its nucleus. Hence volume of nucleus is 10^{-12} times smaller than atom. This concludes that atom is almost empty.
- For measuring atomic mass and its sub particles, new unit of mass is introduced as atomic mass unit '*u*'.

$$1u = \frac{\text{mass of one } \frac{12}{6}C \text{ atom}}{12}$$

$$1u = 1.660539 \times 10^{-27} \text{ kg}$$

- Atomic mass unit is not an integral multiple of *u* due to presence of isotopes (atoms of same element with different atomic masses).
- We have learnt in chapter-12 'atoms' that all mass and positive charges of an atom is concentrated in its centre known as nucleus.
- Chadwick discovered a new sub-particle in nucleus known as neutron. It is neutral in nature. Mass of neutron

$$m_n = 1.00866 \text{ u} = 1.6749 \times 10^{-27} \text{ kg}$$

- The composition of a nucleus can now be described using the following terms and symbols :
  - Z – atomic number = number of protons (also equal to the number of electrons)
  - N = neutron number = number of neutrons
  - A = atomic mass number = (*Z* + *N*) = total number of protons and neutrons
- An atom is represented by ${}^{A}_{Z}X$ where

  X = Symbol of element
- **Isotopes :** Two atoms of an element (*Z* is same) having different atomic mass number (due to the different number of neutrons) are said to be isotopes.
- **Isobars :** Two atoms of different elements having same mass number but different atomic number are said to be isobars.
- **Isotones :** Two atoms of different elements having different mass number and atomic number such that their difference is same are said to be isotones. It means they have same number of neutrons.

- **Size of the nucleus :** A nucleus of mass number A has a radius

  $R = R_0 A^{1/3}$ where, $R_0 = 1.2 \times 10^{-15}$ m

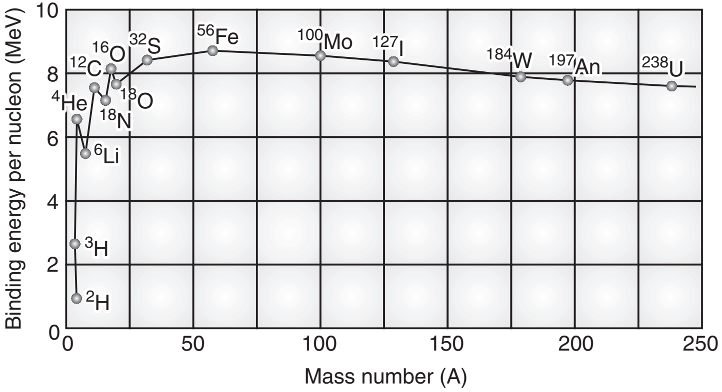
  Nuclear matter density $= 2.3 \times 10^{17}$ kgm^{-3}
- Earlier it was believed that anything in the universe can be classified into matter or radiation. Einstein proposed that there are two form of energy which are interconvertible.

  $E = mc^2$; where, c is speed of light.
- With this relation we may calculate $1 \text{ u} = 931.5$ MeV
- **Mass Defect :** The difference in mass of a nucleus and its constituents, ΔM, is called the mass defect, and is given by

  $$\Delta M = [Zm_p + (A - Z)m_n] - M$$
- **Binding Energy :** Binding energy of a nucleus is that quantity of energy which when given to nucleus, its nucleons will become free and will leave the nucleus. It is having negative sign.

  $E_b = \Delta Mc^2$ where E_b is binding energy.
- **Binding energy per nucleons (B_E/A) :** A more useful measure of the binding between the constituents of the nucleus is the binding energy per nucleon, E_{bn} or E_b/A, which is the ratio of the binding energy E_b of a nucleus to the number of the nucleons, A, in that nucleus.

  $$E_{bn} = \frac{E_b}{A}$$
- **Relation between BE/A and Stability of elements atom**



  - Higher the Binding Energy per Nucleons, more stable is the element. High Binding Energy per Nucleons means we have to supply more energy to free nucleons or difficult to break the nucleons.
  - For most of the atoms $30 < A < 200$. The Binding Energy per Nucleons is fairly constant and quite high and is maximum for A = 56 about 8.75 MeV.
  - For $A < 30$ and $A > 170$; Binding Energy per Nucleons is quite low.
- If a nucleus of lower binding energy is converted into higher binding energy then energy is released.
- There are two methods of converting lower binding energy is converted into higher binding energy.
  - **Fission :** A heavy nucleus (low Binding Energy per Nucleons) is broken into two lower nucleus (higher Binding Energy per Nucleons) and energy is released. This process is known as Fission.

    Example : ${}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{92}^{236}\text{U} \rightarrow {}_{56}^{144}\text{Ba} + {}_{36}^{89}\text{Kr} + 3{}_0^1\text{n} + \text{Energy}$
  - **Fusion :** Two light nucleus (low Binding Energy per Nucleons) are joined and form one nucleus of higher Binding Energy per Nucleons, energy is released. This process is known as Fusion.

    Example : ${}_1^1\text{H} + {}_1^1\text{H} \rightarrow {}_1^2\text{H} + \text{e}^+ + \text{v} + 0.42\,\text{MeV}$
- **Nuclear Force :**
  - The binding energy per nucleon is approximately 8 MeV, which is much larger than the binding energy in atoms.
  - This high Binding Energy per Nucleons is in order to overcome repulsive force between proton and bind both protons and neutrons into the tiny nuclear volume.
  - The nuclear force is much stronger than the Coulomb force acting between charges or the gravitational forces between masses but it's a short range force $\left(\propto \frac{1}{r^7}\right)$

TOPIC-2

Radioactivity and Nuclear Reactor

Quick Review

- **Radioactivity :** When atoms become very heavy, neutrons are unable to bind and some nucleons keep on leaving the nucleus. These atoms/elements are known as radioactive elements and process of spontaneous ejection of nucleons or radiations is known as radioactivity.
- There are three types of radioactive decay in nature.
  - **α-decay :** In this decay α-particles ($^{4}_{2}\text{He}$ nucleus) eject out.
  - **β-decay :** In this decay electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) eject out.
  - **γ-decay :** In this high energy photons are emitted.
- **Properties of α-rays :**
  - α-rays consist of doubly ionised helium atoms.
  - After α-decay a nucleus is transformed into another nucleus.

    Examples :

    $$^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$$

    $$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

  - The difference between the initial mass energy and the final mass energy of the decay products is called the Q value of the process or the disintegration energy. Thus, the Q value of an alpha decay can be expressed as

    $$Q = (m_X - m_Y - m_{He})c^2$$

  - α-rays are deflected by the electric field and the magnetic field.
  - The velocity of α-rays is about 10% of the velocity of light.
  - α-rays affect the photographic plate.
  - In α-decay; mass number of daughter nucleus changes by 4 units and atomic number changes by 2 units.
- **Properties of β-rays :**
  - β-rays consist of electrons or positrons.
  - In beta minus (β^-) decay, an electron is emitted by the nucleus. In this a neutron is converted into proton and electron and electron is emitted out along with antineutrino. Hence there is no change in mass number and +1 change in atomic number.

    Example :

    $$^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \bar{\nu} \quad (T_{1/2} = 14.3\ d)$$

  - In beta plus (β^+) decay, a positron is emitted by the nucleus. In this a proton is converted into neutron and positron is emitted out along with neutrino. Hence there is no change in mass number and –1 change in atomic number.

    Example :

    $$^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ + \nu \quad (T_{1/2} = 2.6\ y)$$

  - In these equations ν, $\bar{\nu}$ are known as neutrino and anti neutrino particles respectively. They have no charge, approximately no mass and unreactive.
  - β-rays are deflected by the electric field and the magnetic field.
  - The velocity of β-rays is up to 90% of the velocity of light.
  - β-rays affect the photographic plate.
- **Properties of γ-rays**
  - There are energy levels in nucleus also.
  - When a excited state nucleus make a transition to a lower energy state, it radiate electromagnetic radiation As the energy differences between levels in a nucleus are of the order of MeV, the emitted photons are of MeV energies and are called gamma rays.

- Most radionuclides after an alpha decay or a beta decay leave the daughter nucleus in an excited state. Then this excited daughter nucleus come to ground state by radiating γ-rays.
- γ-rays are photons of very short wavelength of the order of 10^{-11} m to 10^{-13} m.
- γ-rays carry no charge and hence are not deflected by the electric field and the magnetic field.
- γ-rays move with the speed of light.
- γ-rays affect the photographic plate.

➢ **Laws of Radioactive decay :**

- Rate of decay $\frac{\Delta N}{\Delta t} \propto N$ where, N = number of nuclei in the sample

For very small time interval

$\frac{dN}{dt} = -\lambda N$ where, λ is disintegration constant

By integrating both sides, ...(i)

$$N(t) = N_0 e^{-\lambda t}$$

- Rate of disintegration, Differentiating eq^n (i)

$R = \lambda N_0 e^{-\lambda t}$

or $R = \lambda N$

where, $R = \frac{-dN}{dt}$

Often rate of disintegration R ($= -dN/dt$) is more important than in N itself. It gives us the number of nuclei decaying per unit time.

➢ **Alternative form of law of Radioactive decay:**

$$R = R_0 e^{-\lambda t}$$

where, R_0 is the radioactive decay rate at time $t = 0$, and R is the rate at any subsequent time t.

The SI unit for rate of radioactive decay is becquerel. One becquerel (Bq) is one decay per second.

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ Bq}$$

➢ **Measurement of life of Radionuclide :**

- **Half lifetime ($T_{1/2}$) :** It is the time period in which both N and R reduce half of initial quantity.

$$T_{1/2} = \frac{ln2}{\lambda} = \frac{0.693}{\lambda}$$

➢ **Mean life (τ) :** It is the time at which both N and R have been reduced to e^{-1} of their initial values.

$$\tau = 1/\lambda$$

➢ **Relation between half life time and mean life :**

$$T_{1/2} = \frac{ln2}{\lambda} = \tau \, ln\, 2$$

➢ We may also derive from above formulae of half lifetime and radioactive decay rate that

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \quad \text{where, } n = \frac{t}{T_{½}}$$

➢ **Nuclear Energy by artificial Fission and Fusion process**

- **Fission :** When a heavy nucleus is broken into two smaller nucleus, the process is known as fission. In this process huge amount of energy is released.

  When a neutron was bombarded on a uranium target, the uranium nucleus broke into two nearly equal fragments releasing great amount of energy.
- Some combination of products of above reaction are

$${}^1_0n + {}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U} \rightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^1_0n$$

$${}^1_0n + {}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U} \rightarrow {}^{133}_{51}\text{Ba} + {}^{99}_{41}\text{Nb} + 4{}^1_0n$$

$${}^1_0n + {}^{235}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2{}^1_0n$$

- The energy released (the Q value) in the fission reaction of nuclei like uranium is of the order of 200 MeV per fissioning nucleus.

Nuclear Reactor :

- When $^{235}_{92}\text{U}$ undergoes a fission after bombarded by a neutron, it also releases an extra neutron. This extra neutron is then available for initiating fission of another $^{235}_{92}\text{U}$. Hence this is a chain reaction. Controllable form of chain reaction is used in nuclear reactor.
- Reason for not having a chain reaction in efficient way :
  - Poor concentration of Uranium

    Solution : Enrichment of ore and make uranium rods in low quantity of uranium.
  - Speed of neutron is very high hence they penetrate through Uranium.

    Solution : Convert high energy neutron to thermal neutron by collision with moderator.
  - Controlling the rate of reaction.

    Solution : Insertion of control rods so that rate of reaction K is equal to one.

- **Benefits of Nuclear reactor :**
  - Huge energy from small quantity of uranium. It's a good replacement of coal reactor.
- **Risks of Nuclear reactor :**
  - Uncontrollable blast
  - **Radiation leakage :** Uranium waste is also radioactive.
- **Nuclear Fusion :** Two light nucleus (low Binding Energy per Nucleon) join and form one nucleus of higher Binding Energy per Nucleon, energy is released. This process is known as Fusion.

  Some Examples of nuclear fusion are

$$^{1}_{1}\text{H}+^{1}_{1}\text{H}\rightarrow ^{2}_{1}\text{H}+e^{+}+\nu+0.42\,\text{MeV}$$

$$^{2}_{1}\text{H}+^{2}_{1}\text{H}\rightarrow ^{3}_{2}\text{He}+n+3.27\,\text{MeV}$$

$$^{2}_{1}\text{H}+^{2}_{1}\text{H}\rightarrow ^{3}_{1}\text{H}+^{1}_{1}\text{H}+4.03\,\text{MeV}$$

- Stars have fusion energy.
- Fusion process gives more energy than fission process. In the above examples of fusion and fission, energy from one unit mass by fusion is 6.7 MeV while from fission it is 1 MeV
- **Advantages of Nuclear fusion reactor :**
  - It's a clean fuel. No radioactive wastage in this process.
  - Hydrogen is available in plenty.
- **Problems of Nuclear fusion reactor :**
  - Cannot be stopped unless the whole stock is burnt.
  - Storage of hydrogen plasma.
- Hydrogen bomb is uncontrollable nuclear fusion reaction.
- **Thermal nuclear fusion reaction in Sun :** Fusion of hydrogen nuclei into helium nuclei is the source of energy of all stars including our sun.

$$^{2}_{1}\text{H}+^{1}_{1}\text{H}\rightarrow ^{2}_{1}\text{H}+e^{+}+\nu+0.42\text{ MeV} \quad \text{...(i)}$$

$$e^{+}+e^{-}\rightarrow \gamma+\gamma+1.02\text{ MeV} \quad \text{...(ii)}$$

$$^{2}_{1}\text{H}+^{1}_{1}\text{H}\rightarrow ^{3}_{2}\text{He}+\gamma+5.49\text{ MeV} \quad \text{...(iii)}$$

$$^{3}_{2}\text{H}+^{3}_{2}\text{H}\rightarrow ^{4}_{2}\text{He}+^{1}_{1}\text{H}+^{1}_{1}\text{H}+12.86\text{ MeV} \quad \text{...(iv)}$$

The combined effect of above reactions is

$$4^{1}_{1}\text{H}+2e^{-}\rightarrow ^{4}_{2}\text{He}+2\nu+6\gamma+26.7\,\text{MeV}$$

❑❑

UNIT - IX : Electronic Devices

Chapter - 14 : Semiconductors Electronics : Materials, Devices and Simple Circuits

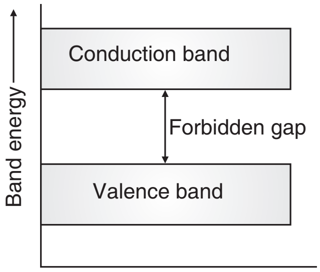
TOPIC-1

Energy Band

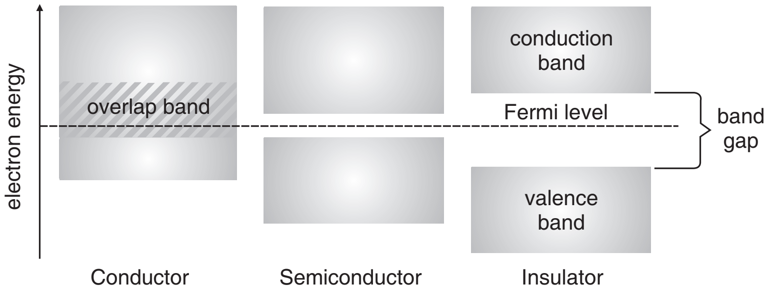
Quick Review

Energy bands :

- Electrons in same orbit exhibiting different energy levels known as energy band.
- Energy bands consist of large number of closely spaced energy levels that exists in crystalline materials.
- The bands are collection of individual energy levels of electrons that surrounds every atom.
- In solids, there are three important energy bands such as Valence band, Conduction band, Forbidden band or forbidden gap.

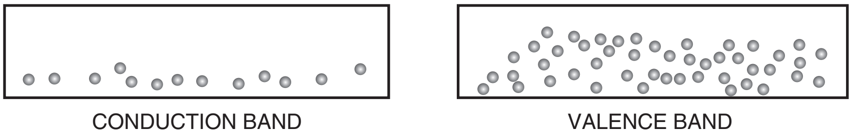


- The collections of energy levels of free electron which move freely around the material are called as conduction band.
- There is an extra energy required to conduction band by valence electrons to move, which is known as Forbidden energy.
- The collections of energy levels which are partially or wholly filled are known as valence band.
- In this, wave functions of every electrons overlap with those of electrons confined to neighbouring atoms.
- The energy associated with forbidden band is known as energy gap which is measured in electron volt (eV) where, 1 eV = 1.6×10^{-19} J.
- The energy band theory is classified as Conductors, Insulators and Semiconductors.

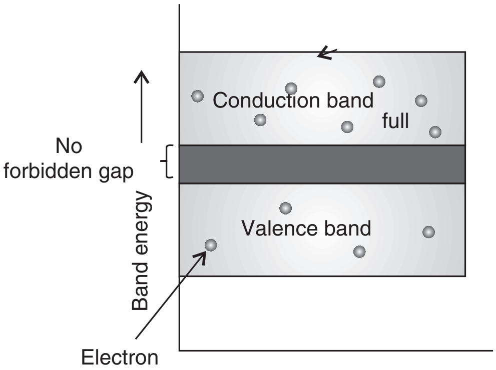


Energy bands in Conductors

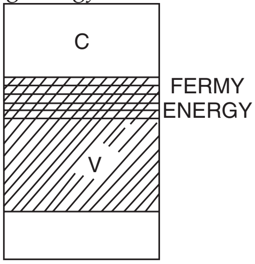
- Conductors are the materials where the conduction band and valence band gets overlapped.



- The overlapping of conduction and valence bands without energy gap forms a conduction band.

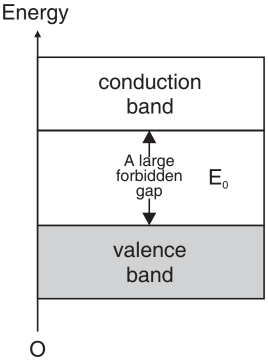


- In this, an electron that receives any acceptable low energy will able to move freely among the bands.
- In conductors, the last occupied band of energy levels is only partially filled and electrons occupy lowest levels one by one as per Pauli's exclusion principle.
- Here, electrons of valence band moves freely in partially filled conduction band.
- The highest energy level occupied at absolute zero temperature by electrons in partially filled conduction band is known as Fermi level while corresponding energy is known as Fermi energy.

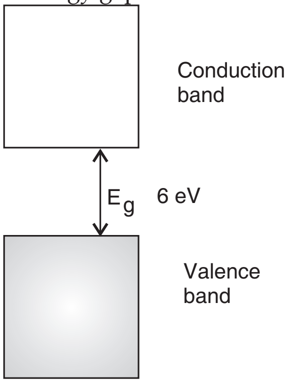


Energy bands in insulators

- Insulator or dielectric is a material where conduction band and valence band does not overlapped.

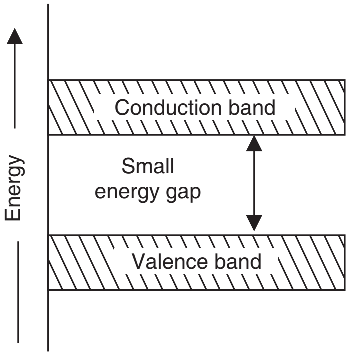


- In this band, the distance between conduction band and valence band exceeds by 3 eV as electron transfer from valence band to conduction band needs more power.
- Due to requirement of more power, insulators conduct practically no electric current.
- In insulators, empty conduction band and full valence band have energy gap of around 6 eV.
- Here, the band which separates conduction band and valence band is known as Forbidden band.
- Example of an insulator is diamond with energy gap of around 5.4 eV.

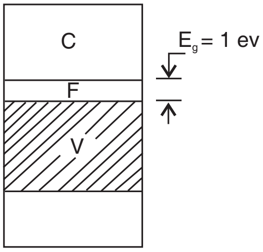


Energy bands in Semiconductor

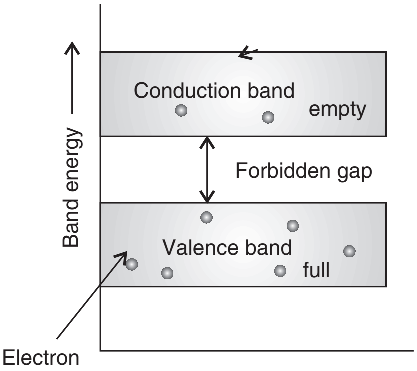
- Semiconductors are materials in which, conduction band and valence band are not overlapped.



- The distance between conduction band and valence band is around 0.1 to 2 eV as electron transfer from valence band to conduction band needs less energy.



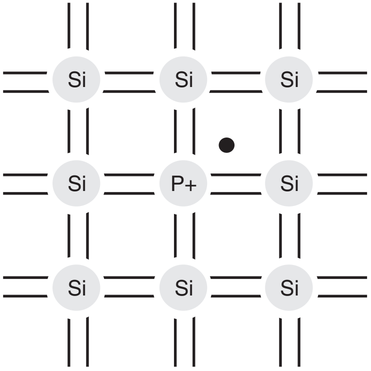
- In such materials, the energy provided by the heat at room temperature is sufficient to lift the electrons from the valence band to the conduction band.



- In this, the band that separates conduction and valence band is known as Forbidden band.
- Semiconductors will behave as insulators at 0K as no electrons are available in conduction band.
- Examples of semiconductors are Silicon (14) and Germanium (32) having energy gaps as 1.12 eV and 0.75 eV respectively.

Doped Semiconductors

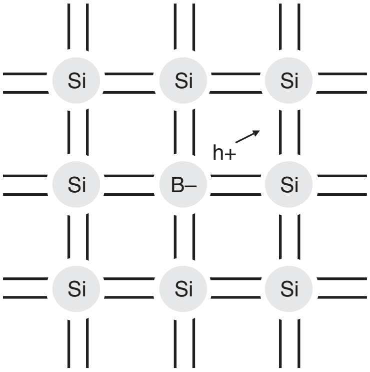
- Doping is adding of impurities to silicon crystal lattice so as to increase the number of carriers.
- For raising electric conductivity, semiconductors are mixed with either pentavalent impurity such as Antimony (Sb), Arsenic (As) or trivalent impurity such as Indium (In), Gallium (Ga).



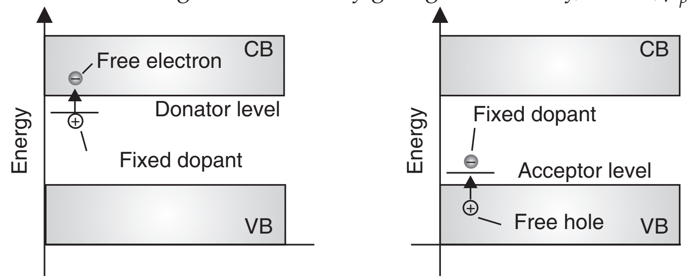
- By adding small number of atoms will help in increasing either the number of electrons or holes.
- If Phosphorous with 5 valence-band electrons are added, it will donate an extra e^- which will freely move around

and leaves a positive charge nucleus.

- The crystal will be electrically neutral known as "*n*-type" material with negative carriers where concentration of donor atoms is 10^{15} cm^{-3} ~ 10^{20} cm^{-3} having mobility $\mu_n \approx 1350$ cm^2/V
- If Boron atom with 3 valence band electrons are added, it will accepts e^- and gives extra holes (h^+) to move freely which leaves behind negatively charged nucleus.



- The crystal is electrically neutral known as "*p*-type" silicon in which concentration of acceptor atoms ~10^{28} cm^{-3} where hole movement needs breaking of bond thereby giving low mobility, where, $\mu_p \approx 500$ cm^2/V



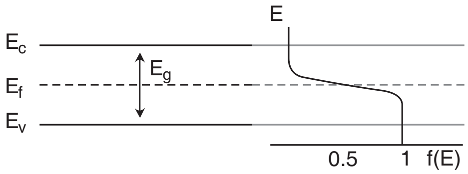
n-semiconductor p-semiconductor

Energy plotted as a function of position

E_C Conduction band
Lowest energy state for a free electron
Electrons in the conduction band means current can flow

E_V Valence band
Highest energy state for filled outer shells
Holes in the valence band means current can flow

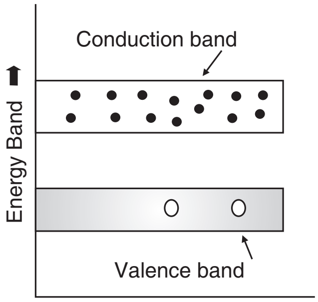
E_f Fermi Level
Shows the likely distribution of electrons



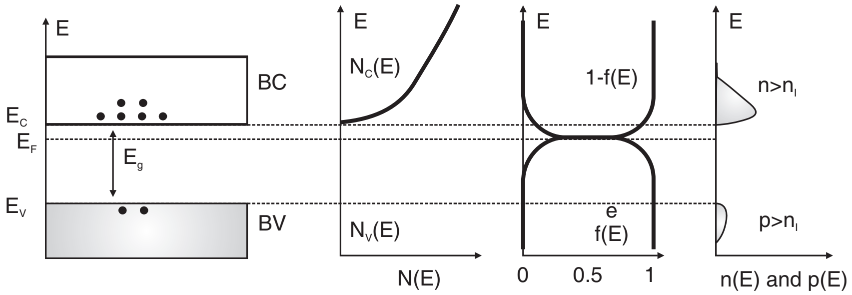
E_G Band gap
Difference in energy levels between E_C and E_V
No electrons (e^-) in the bandgap (only above E_C or below E_V)
$E_G = 1.12$ eV in Silicon

Energy Diagram of N-Type Semiconductor

- On doping a semiconductor with pentavalent impurity like antimony (Sb) or arsenic (As), extrinsic semiconductor so obtained is known as *n*-type.
- *n*-type semiconductor has large number of free electrons known as majority (charge) carriers and small number of holes known as minority (charge) carriers.
- Impurity atom in *n*-type semiconductor is called donor which generates new energy level below the conduction band.
- In *n*-type material there are electrons energy levels near the top of the band gap so that they can be easily excited into the conduction band.
- With this, effective fermi level gets shifted to a point about halfway between the donor levels and conduction band.



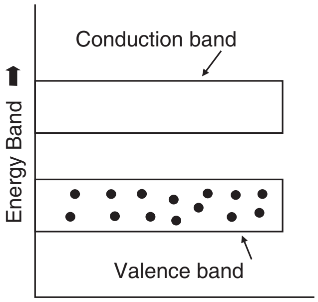
- Electrons get elevated to conduction band with energy given by applied voltage that moves through the material.
- Addition of Pentavalent impurity results in large number of free electrons.
- When thermal energy is imparted to semiconductor, hole-electron pair appears give rise to minute quantity of free electrons which leaves the hole behind in valence band.



- In *n*-type, when donor density increases, Fermi level moves closer to the edge of conduction band.
- If $N_D = N_C$ then Fermi level enters the conduction band and degenerates the semiconductor.

Energy Band Diagram of *p*-Type Semiconductor

- On doping a semiconductor with trivalent impurity like indium (In) or gallium (Ga), extrinsic semiconductor so obtained is known as *p*-type.



- *p*-type semiconductor has large number of holes known as majority (charge) carriers where number of free electrons is small known as minority (charge) carriers.
- Impurity atom in *p*-type semiconductor is known as acceptor atom.
- In *p*-type, extra holes in band gap allow excitation of valence band electrons which leaves mobile holes in valence band.
- Large number of holes in covalent bond is created in crystal with trivalent impurity.
- Here, a small amount of free electrons is available in conduction band which are produced by imparting thermal energy to germanium crystal that develops an electron-hole pairs.
- In this, holes are more in number as compared to electrons in conduction band due to predominance of holes over electrons making the material to act as *p*-type.
- In *p*-type, when the acceptor density increases, Fermi level moves closer to the edge of the valence band.
- If $N_A = N_v$ then Fermi level enters the valence band and degenerates the semiconductor.

Properties of Semiconductors :

(i) Semiconductors have negative temperature coefficient of resistance, *i.e.*, their resistance decreases with the rise in temperature.

(ii) The conductivity of semiconductors changes appreciably when a suitable trivalent or pentavalent impurity is added to them.

(iii) The difference of energy between the conduction and the valence bands is known as forbidden energy gap (E_g). Its value differs for conductors, semiconductors and insulators.

For conductors, E_g may be negligibly small or zero. For insulators, it may be 5 to 15 eV and for semiconductors, E_g is of the order of 1 eV.

(iv) At the absolute zero, all the lower energy levels of the valence band are completely filled. The maximum energy that an electron in the valence band possess at absolute zero is called Fermi energy and the level corresponding to it is called Fermi energy level.

(v) As the temperature rises, many electrons from the valence band jump to the conduction band, then a vacancy is created in the valence band. This vacancy is known as a *hole*. As the sample is electrically neutral, the number of free electrons in the conduction band is equal to the number of holes in the valence band.

- At absolute zero, a semiconductor behaves as a perfect insulator.
- The doping of semiconductor with small amount of impurity, drastically increases their conductivity.
- At low temperature, the free electrons are present in the valence band of the semiconductor. As the temperature rises, the electrons cross over to the conduction band. The fraction of electrons in the conduction band at temperature T (kelvin) is given by

$$f = f_0 e^{-E_g/kT}$$

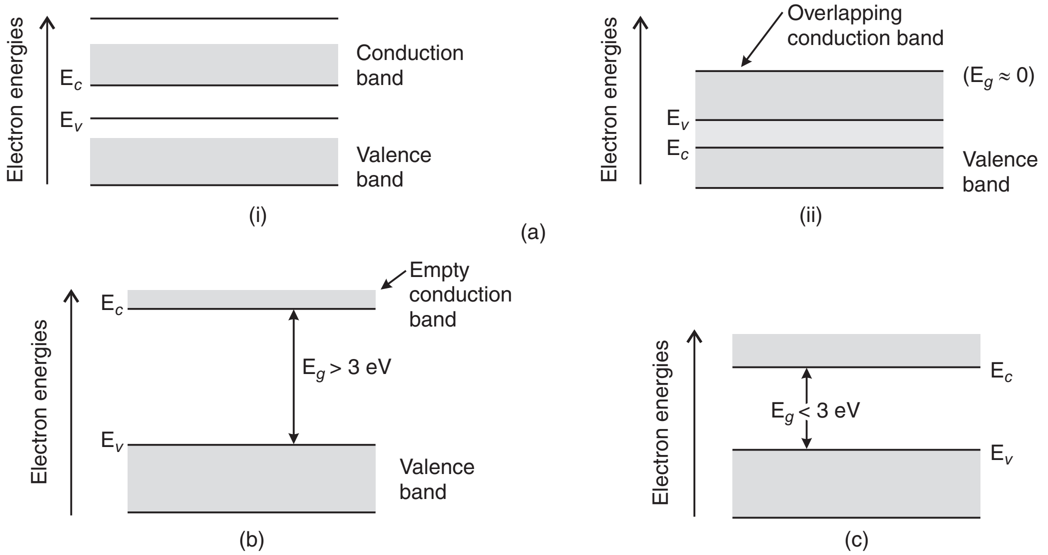
where, E_g = forbidden band energy gap and k is the Boltzmann constant.

- Let n_e = number density of free electron in the conduction band, n_h = number density of holes in the valence band and n_i = number density of the intrinsic charge carriers.

  For intrinsic semiconductors : $n_e = n_h = n_i$

  For doped semiconductor : $n_e \times n_h = n_i^2$

Comparison of (a) metals (b) insulators and (c) semiconductors as per energy bands



(a) metals, (b) insulators and (c) semiconductors.

Intrinsic semiconductor

- A pure semiconductor is called an intrinsic semiconductor.
- Electrons and holes in intrinsic semiconductors are called intrinsic charge carriers.

Extrinsic semiconductor

- The semiconductor is called extrinsic semiconductor where one impurity atom is added for about 10^8 atoms of semiconductor.
- Electrons and holes generated in the extrinsic semiconductors are called extrinsic charge carriers.

Know the Terms

- **Energy band :** Range of energies that an electron may possess in an atom.
- **Valence Band :** Range of Energy possessed by valence electrons.
- **Conduction Band :** Range of energy possessed by electrons.
- **Forbidden Band :** Energy band in between the conduction band and valence band.

TOPIC-2
Semiconductor Diodes and Applications

Quick Review

Semiconductor Material

- Intrinsic semiconductor material are those in which semiconducting properties of material occurs naturally.

- Extrinsic semiconductor materials are those that are manufactured by us to make material behave in required manner.

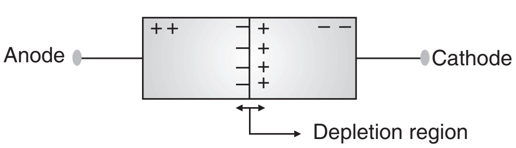
Diode

- Diode is an electronic device consisting of *p*-type and *n*-type junctions.

- The terminal connected to *p*-type crystal is known as anode while the terminal connected to the *n*-type crystal is known as cathode.
- Diode is a semiconductor device that is doped in different proportions as *p* and *n* types which forms the *p-n* junction.
- It is an electrical device that allows the current to move through it in one direction.
- The most common kind of diode in modern circuit design is the semiconductor diode.
- In diodes, current is directly related to voltage as resistor and not like capacitors, where current is related to time derivative of voltage or inductors where derivative of current is related to voltage.
- In diodes the current is not linearly related to voltage, like in a resistor.
- Diodes only consume power and not produce power like a battery and are said to be passive devices.

Semiconductor diode

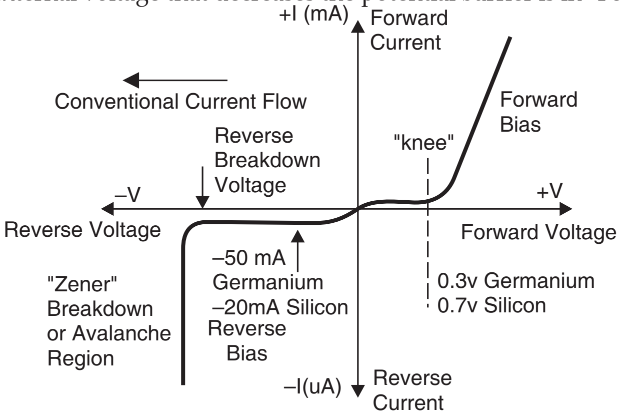
- Semiconductor diodes were first semiconductor electronic devices which is common type of diode that is made of crystalline piece of semiconductor material with *p–n* junction across its terminals.



- When a *p*-type semiconductor material is suitably joined to *n*-type semiconductor the contact surface is called a *p-n* junction.
- The *p-n* junction is also called as semiconductor diode.
- There are many types of semiconductor diode such as : Avalanche diodes, Gunn diodes, light-emitting diodes (LED's), Photodiodes etc.
- Semiconductor diode can be made either from Silicon or Germanium and each differs in size and properties.

I-V characteristics of Diode

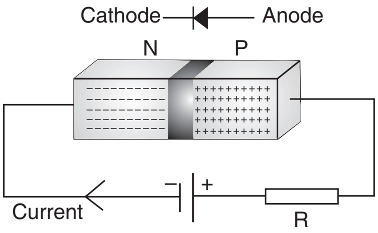
- In I-V characteristics of diode, on voltage axis, "Reverse Bias" is an external voltage potential that increases the potential barrier while external voltage that decreases the potential barrier is in "Forward Bias" direction.



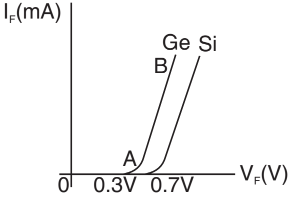
- Biasing in Diode can be Forward Biasing or Reverse Biasing.
- In this, when a small voltage is applied to diode in forward direction, current flows easily.
- Due to certain resistance in diode, voltage will drop slightly as current flows through the diode.
- A typical diode causes a voltage drop of about 0.6 V – 1 V.
- The voltage drop is taken into consideration where many diodes in series are used in the circuit.

Forward Bias

- When an external voltage is applied, where negative of battery is connected to *n* side while positive of battery is connected to *p* side, then voltage potential will get reduced and more current can flow across the junction that decreases the *pn* junction diode width.



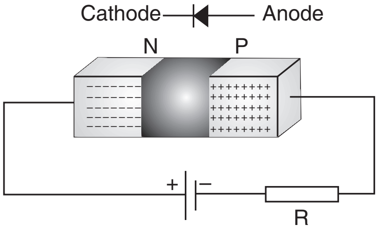
- The positive terminal of battery repels majority carriers, holes in *p*-region while negative terminal repels electrons in *n*-region which pushes them towards the junction.
- Here, an increase in concentration of charge carriers near the junction is observed, where recombination takes place thereby reducing width of depletion region.



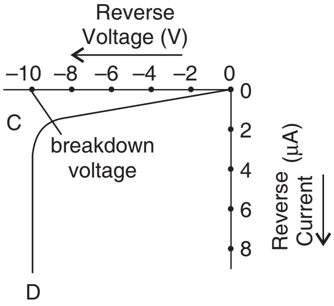
- Due to rise in forward bias voltage, depletion region will continue to reduce its width which results in more and more carriers recombination.

Reverse Bias

- If an external voltage is applied in reverse direction where positive of battery is connected to *n* side while negative of battery is connected to *p* side, then barrier potential will increase and minority charge carriers will flow across junction.



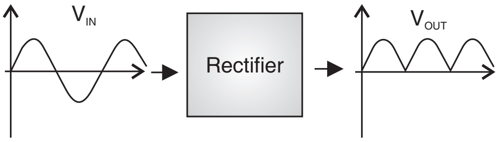
- In this, the current will be quiet small and is independent of external voltage.
- Beyond certain voltage, diode will break down with Avalanche breakdown mechanism or Zener breakdown mechanism.
- Here, negative terminal of battery will attract majority carriers, holes in *p*-region and positive terminal attracts electrons in *n*-region which pulls them away from the junction.
- As a result of this, there will be decrease in concentration of charge carriers near junction which increases the width of depletion region.



- A small amount of current flow due to minority carriers known as reverse bias current or leakage current flows and with rise in reverse bias voltage, depletion region continues to increase in width without any flow of current.

Diode as rectifier

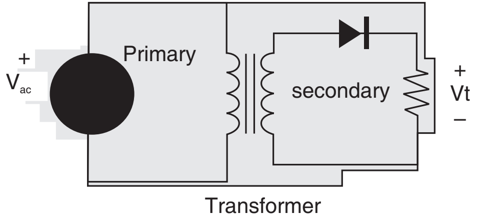
- Rectifier is a circuit which converts *ac* supply into unidirectional *dc* supply.



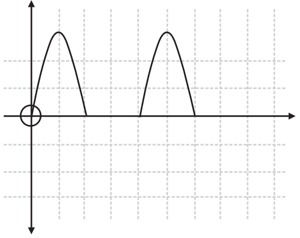
- With rectification, alternating current (AC) gets converted to direct current (DC) through a rectifier.
- The bridge rectifier circuits uses semiconductor diode for converting AC as it allows the current to flow in one direction only.
- A rectifier diode is a two-lead semiconductor that allows current to pass in only one direction.
- It is an ability of the diode to allow the current to flow on only one direction, which is the main operation of every rectifiers.
- Rectifiers can be half wave rectifiers or full wave rectifiers.

Half wave rectification

- Rectification is one of the most famous applications of diode where alternating current (*ac*) gets converted into direct current (*dc*) using one way flow of electrons.



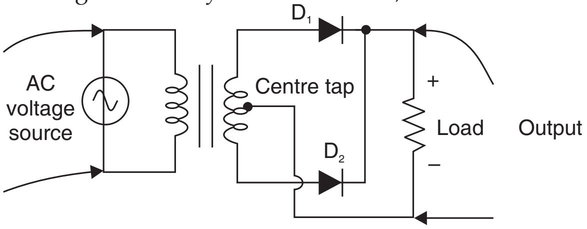
- Rectification reduces power to resistive load.
- The half-wave rectifier with single diode, allows current to flow in one direction.
- Here, *ac* power source V_{ac} is connected to primary side of transformer while secondary terminals of transformer are connected to diode and resistor in series.
- If V_{ac} is in positive cycle, a positive voltage is produced on secondary side of transformer.
- The positive voltage will forward bias the diode and diode will start passing the current as a result, the voltage will drop across the load.
- If V_{ac} is in negative cycle, then secondary side will have negative voltage where diode is reverse biased and will not pass any current.
- With this, voltage drop over load will be zero whose voltage waveform over load resistor looks as shown, where positive side of sinusoidal cycle is present while negative side sinusoidal cycle has been clamped off.



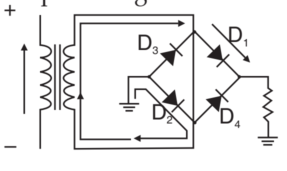
- The output voltage V_t is similar to the output of battery which is always positive.
- The positive waveform is bumpy as single diode is applied to produce half wave rectification where one half of AC wave is removed that will not pass through the diode.

Full wave rectification

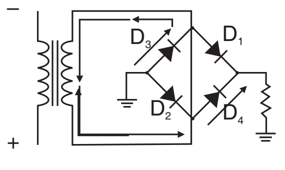
- For rectifying AC power for using both half cycles of sine wave, full wave rectification is used.



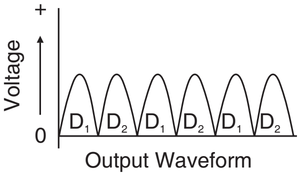
- A simple kind of full wave rectifier is center tap which are used in transformers with two diodes.
- In full wave rectification, in first half-cycle, when source voltage polarity is positive (+) on top and negative (-) on bottom, then only top diode will conduct while bottom diode blocks the current where first half of sine wave is positive on top and negative on bottom.



Current flow positive half-cycle



Current flow negative half-cycle

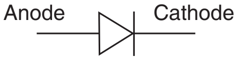


Output Waveform
Time ⟶

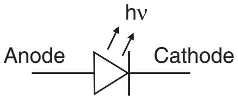
- In the next half-cycle, *AC* polarity gets reverses and other diodes with other half of transformer's secondary winding will conduct current.
- In positive half of input *AC*, diodes D_1 and D_2 are forward biased while diodes D_3 and D_4 are reverse biased which makes load current flows through D_1 and D_2 diodes.
- In negative half cycle of input *AC*, diodes D_3 and D_4 are forward biased while diodes D_1 and D_2 are reverse biased which makes load current to flow through D_3 and D_4 diodes.
- Full-wave rectifiers are used in power supplies for converting AC voltages to DC voltages.

Special purpose *p-n* junction diodes

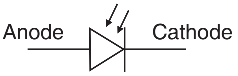
Apart from simple *pn* junction diodes, there are many more types of diodes which are used in various specific applications that take advantage of the behaviour and features.



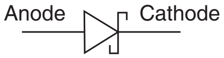
Diode



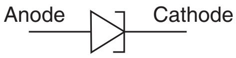
Light-emitting diode



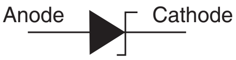
Photodiode



Schottky diode



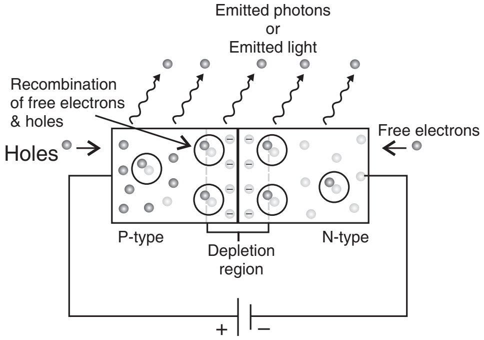
Tunnel diode



Zener diode

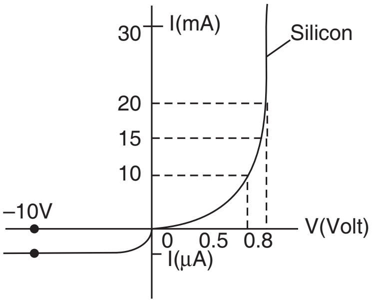
LED

- Light Emitting Diode or LED is most widely used semiconductor diodes among all the different types of semiconductor diodes available today.
- It emits visible light or invisible infrared light when forward biased.
- The LEDs which emit invisible infrared light are used for remote controls.
- LED is light source that uses semiconductors and electroluminescence to create light.
- There are two main types of light emitting diodes: LED and OLED
- LED is different than EL lamp as it uses a small semiconductor crystal with reflectors and other parts to make light brighter and focused in single point.
- OLED is similar to EL lamp, as it uses flat sandwich of materials but is different than LED and EL lamp as it uses carbon molecules in layer that emits light.
- In this, diode in forward biased will make electrons & holes to move fast across the junction and helps in combining constantly by removing one another.
- Electrons which move from *n*-type to *p*-type silicon will combine with holes and disappears leading to atom with more stability giving little burst of energy.



- Recombination of electrons and holes in depletion region decreases the width of the region which allows more charge carriers to cross the *p-n* junction.
- Here, some of the charge carriers from *p*-side and *n*-side will cross the *p-n* junction before they recombine in depletion region.

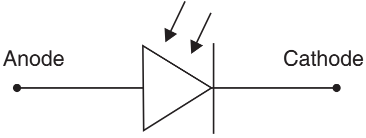
I-V characteristics



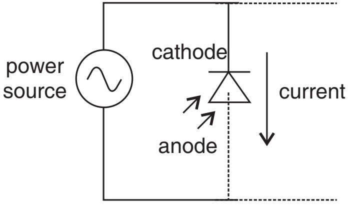
Photodiode

- Photodiode is a transducer which takes light energy and converts it to electrical energy.

- It creates electricity when exposed to light.
- It conducts electric current in similar proportion as the amount of light which falls on it.
- Photo diode has two terminals; anode and cathode.
- It is a *pn* junction which consumes light energy to generate electric current.

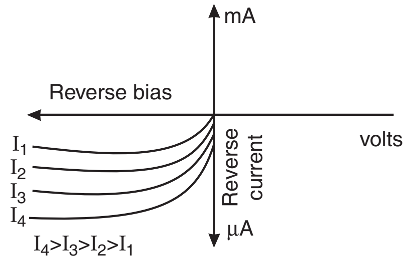


- It is referred to as photo-detector, photo-sensor or light detector.
- It is specially designed to operate in reverse bias condition where *p*-side is connected to negative terminal of battery and *n*-side connected to positive terminal of battery.
- It is sensitive to light as when light or photons fall on it, it easily converts light into electric current.
- Photo diodes can be *pn* junction photo diode, Avalanche photo diode or PIN photo diode.



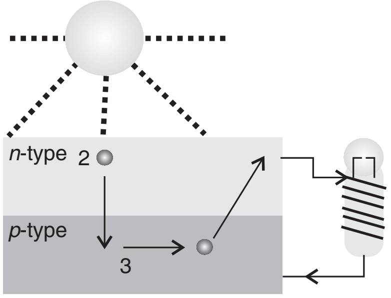
- Photodiode operates in reverse bias in a circuit where anode gets connected to the ground of a circuit and cathode gets connected to positive voltage supply of the circuit.
- In photodiode circuit, current flows from the cathode to anode when exposed to light.
- Photodiode is capable of converting light energy to electrical energy and can be expressed as a percentage known as Quantum Efficiency (Q.E.).

I-V characteristics



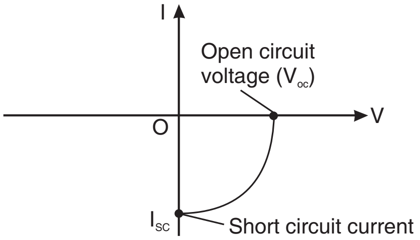
Solar cell

- Solar cell is an electronic device which catches sunlight and converts it into electricity.
- It is compact in size and is bundled with larger units for making solar panels.
- It is a sandwich of two different layers of silicon that are doped for allowing electricity to flow through them in a particular direction.
- In solar cell, lower layer is doped so that it will have less electrons known as *p*-type or positive-type silicon while upper layer is doped to give more electrons known as *n*-type or negative-type silicon.
- In this, there are *n*-type silicon and *p*-type silicon layers that generates electricity using sunlight to make electrons in order to jump across the junction between different types of silicon material.



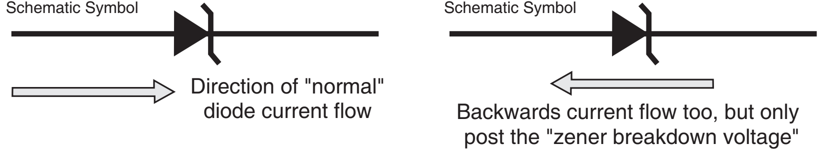
- When sunlight shines on solar cell, photons bombard the upper surface and carry their energy down through the cell to *p*-type layer.
- Such energy is used by electrons to jump across the barrier into upper, *n*-type layer and can be escape out into the circuit.
- By flowing around the circuit, electrons will help in lightening the lamp.

I-V characteristics

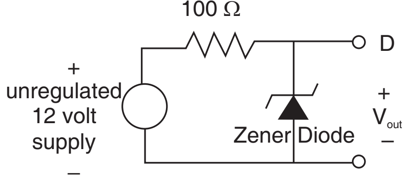


Zener diode and its characteristics

- Zener Diode is an electronic component which can be used to make very simple voltage regulator circuit.



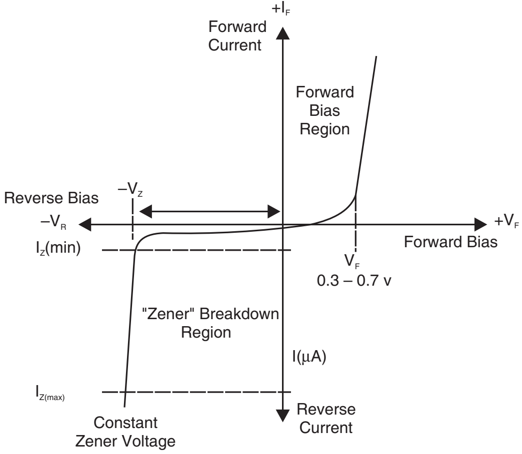
- They are special type of semiconductor diodes which allow the current to flow in one direction when exposed to high voltage.
- It is a *p-n* junction semiconductor device which is designed to operate in reverse breakdown region.
- The breakdown voltage of zener diode is set by controlling the doping level.
- Zener Diode allows electric current in forward direction similar to normal diode and also allows electric current in reverse direction when applied reverse voltage is more than zener voltage.
- It is always connected in reverse direction since it is specifically designed to work in reverse direction.
- Zener Diode circuit enables a fixed stable voltage to be taken from an unstable voltage source like battery that fluctuates as per state of charge of battery.



- It is a diode whose breakdown voltages have been designed to sit at particular voltage level.
- The circuit of Zener Diode has a resistor in series with diode which limits the output current.
- In Zener Diode, there are two breakdown mechanisms : Zener Breakdown or Avalanche Breakdown mechanism.

I-V characteristics

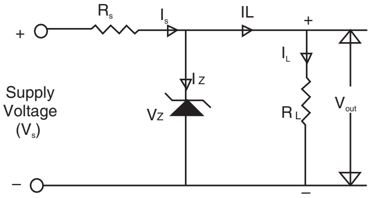
- I-V Characteristics Curve of zener diode shows current-voltage relationship.



- In I-V curve, in right half side, zener diode receives forward voltage which is positive voltage across its anode to cathode terminals.
- In right half side of zener diode characteristics curve, diode is forward biased and current is more.
- In left half side of I-V curve, zener diode will receive positive voltage across its cathode to anode terminals where diode is reverse biased.
- At reverse voltage, current will be very small which is known as leakage current that flows through the diode.
- After hitting breakdown voltage, avalanche current will sharply increase.
- At breakdown voltage point, when voltage of zener diode reaches, it remains constant inspite of increase in current making zener diode suitable for voltage regulation.

Zener diode as voltage regulator

- Voltage regulation is a measure of ability of circuit to maintain constant voltage output under variation either in input voltage or load current.
- In zener diode voltage regulator circuit :
  - resistor R_s is used to limit reverse current through diode to safer value.
  - V_s and R_s are selected such that diode operates in breakdown region.
  - series resistor R_s absorbs output voltage fluctuations to maintain voltage across load to constant value.



- Zener diode maintains constant voltage across load as long as supply voltage is more than zener voltage.
- When input voltage increases, current through Zener diode also increases keeping the voltage drop constant.
- Current in the circuit increases the voltage drop across the resistor which increases by an amount equal to difference between the input voltage and zener voltage of the diode.

Know the Formulae

- $I = I_S + I_O{*}(1 - e^{V/(\eta * Vt)})$

$$\text{NEP} = \frac{\text{noise current (A)}}{\text{responsivity (A/W)}}$$

- The diffusion current during forward bias is given by

$$I_{di} = I_0 \exp.\left(\frac{eV}{kT}\right),$$

where I_0 is the minimum value of the diffusion current corresponding to the forward bias, $V = 0$.

- The diffusion current during the reverse bias is given by

$$I_{di} = I_0 \exp.\left(\frac{-e|V|}{kT}\right)$$

where I_{di} is the value of the diffusion current when the junction diode is unbiased. That is the absolute value of the bias, $|V| = 0$.

- Efficiency of a diode rectifier is given by

$$\eta = \frac{\text{Output d.c. power}}{\text{Input a.c. power}} \times 100$$

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100$$

- For a half wave rectifier :

$$\eta = \frac{40.6}{\left(1 + \frac{r_f}{R_L}\right)}$$

where r_f is the forward bias resistance.

For $r_f = R_L$, $\eta = 20.3\%$.

and for $r_f << R_L$, $\eta = 40.6\%$.

- For a full wave rectifier

$$\eta = \frac{81.2}{\left(1 + \frac{r_f}{R_L}\right)}$$

For $r_f = R_L$, $\eta = 40.6\%$.

and for $r_f << R_L$, $\eta = 81.2\%$.

- Ripple factor

$$\gamma = \frac{I_{ac}}{I_{dc}} = \left[\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1\right]^{\frac{1}{2}}$$

$$= \frac{r.m.s.\ \text{value of fluctuating current}}{\text{average } d.c.\ \text{value of current}}$$

The ripple factor for half-wave rectifier is 121% and that for full-wave rectifier is 48.2%.

- The root mean square current of rectifier :

  For a half-wave rectifier : $I_{rms} = \frac{I_0}{2}$

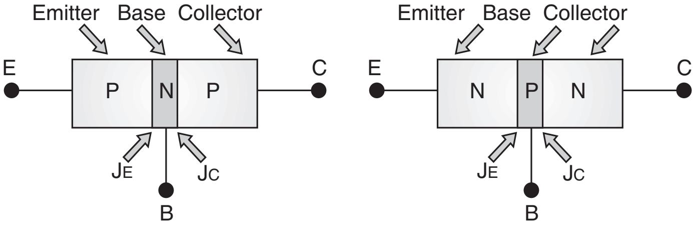
  For a full-wave rectifier : $I_{rms} = \frac{I_0}{\sqrt{2}}$

TOPIC-3
Transistors and its Applications

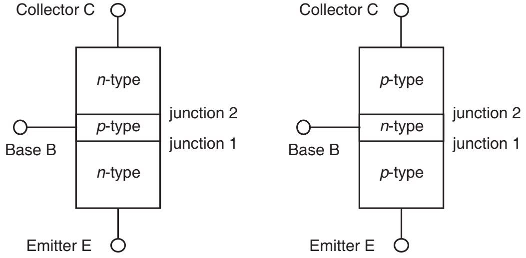
Quick Review

Junction transistor

- A junction transistor is a bipolar transistor having two *p-n* junctions that combines to form either an *n-p-n* or *p-n-p* transistors with three electrodes; emitter, base and collector.



- In junction transistor there is a small current flowing in base which causes larger current to flow in collector.
- In this, when a small current in the base increases, there will be more current in the collector which will be upto maximum of saturation value.
- If the current in the base decreases to zero, the current in the collector will be zero.
- BJT i.e., Bipolar junction transistor consists of three-layer that are sandwich of doped extrinsic semiconductor materials such as *p-m-p* or *n-p-n*, where each layer forms a transistor where each layer shows wire for connection.

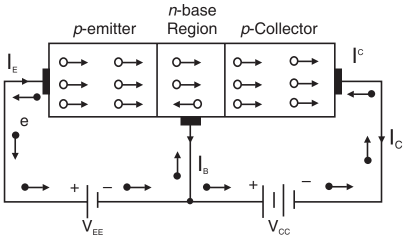


npn-transistor *pnp*-transistor

- In *npn* transistor, a thin and lightly doped *p*-type base is sandwiched between heavily doped *n*-type emitter and *n*-type collector
- In *pnp* transistor, a thin and lightly doped *n*-type base is sandwiched between heavily doped *p*-type emitter and *p*-type collector

Transistor action

- *n-p-n* and *p-n-p* transistors behave in same way except change in biasing and major charge carriers.
- **In *p-n-p* transistor :**

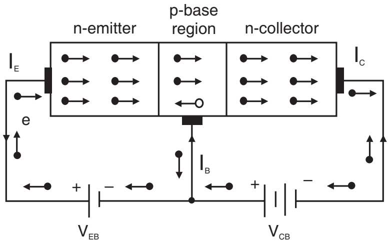


- Conduction is done by holes while in *n-p-n* transistor, conductio is done by electrons.

- Forward bias causes electrons in *n*-type emitter to flow towards the base which constitutes emitter current I_E.
- Electrons while flowing through *p*-type base will tend to combine with holes as base is lightly doped and thin where few electrons on combining with holes forms the base current I_B while remaining electrons crosses the collector region to form collector current I_C.
- Emitter current I_E will be equal to sum of collector current I_C and base current I_B, $I_E = I_B + I_C$

➢ **In *n-p-n* transistor :**

- Conduction is done by electrons.
- Collector, emitter and base are shorted together and forms the two depletion regions that surrounds the base.
- Diffusion of negative carriers in base and positive carriers out of base shows electric potential.
- When transistor is biased for normal operation, base terminal will be positive with respect to emitter with positive collector.

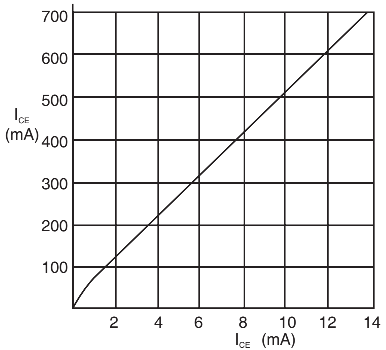


Characteristics of Transistor

➢ A bipolar junction transistor operates in Common Base (C_B) mode, Common Emitter (C_E) mode and Common Collector (C_C) mode.

➢ In a graph of I_{CE} vs I_{BE} , transfer characteristics of transistor and slope shows that current gain h_{fe} with other parameters showing performance of transistor.

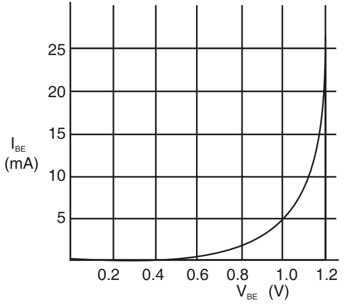
➢ The graph shows BJTs characteristics which are used as voltage amplifier where very low gain results transistor with current gain of 20-50 while high gain ranges from 300 – 800.



➢ In a graph of base emitter current I_{BE} against base emitter voltage V_{BE} shows input conductance of transistor, the curve is used to find the input resistance of a transistor.

➢ Here, sharpness of curve when V_{BE} is above 1 volt shows very high input conductance with large increase in current for small increase in V_{BE}.

➢ Also, input resistance is low and ranges from 0.6 to 0.7 volts showing variance in input resistance of a transistor as per flow of base current.

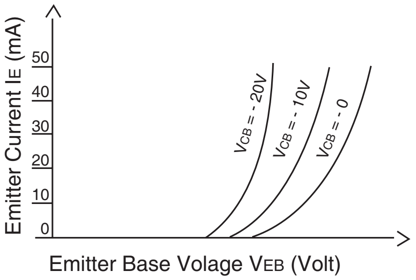


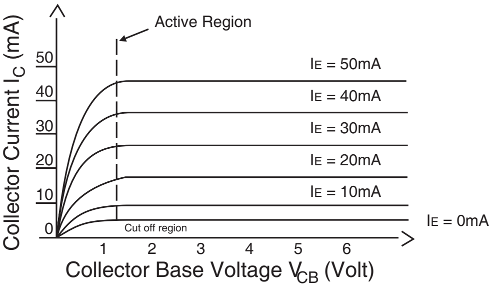
Characteristics of *BJT* in Common Base

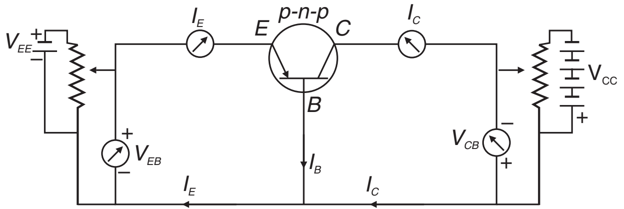
➢ In a *BJT* input characteristics of *p-n-p* transistor in common base, input current is emitter current (I_E) while input voltage is collector base voltage (V_{CB}).

➢ In this, when emitter base junction is forward biased, graph of I_E vs V_{EB} is same for forward bias characteristics of *p-n* diode where I_E increases for fixed V_{EB} when V_{CB} increases.

➢ In *p-n-p* transistor, I_E and V_{EB} are positive while I_C, I_B, V_{CB} are negative which describes three regions in the curve; active region, saturation region and cut off region.



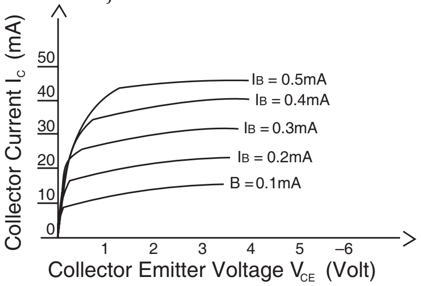




Common base transistor

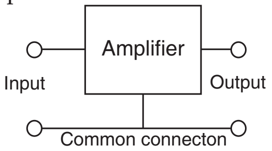
Characteristics of *BJT* in Common Emitter

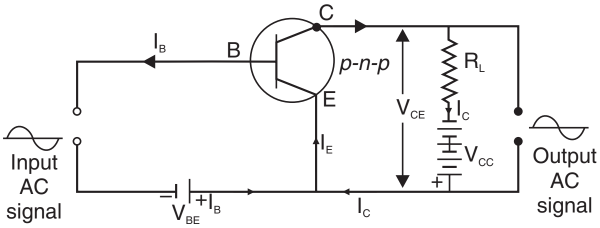
- In input characteristic of *CE* configuration, I_B is input current while V_{BE} is input voltage and relation exists between I_B and V_{BE} with V_{CE}.
- In this, *CE* input characteristics are similar to forward biased of *p-n* diode where decrease in base width appears with increase in V_{CB}.
- In output characteristic for *CE* mode, graph shows relation collector current (I_C) and collector - emitter voltage (V_{CE}) with base current I_B where CE mode shows active region, cut-off region and saturation region.
- Active region has collector region reverse biased and the emitter junction forward biased.
- Cut-off region has emitter junction reverse biased and the partial cut-off of collector current.
- Saturation region has collector and emitter junction forward biased.



Transistor as Amplifier (Common Emitter Configuration)

- Amplifier is an electronic device that increases the strength of weak signal.
- In an amplifier, when an input of low amplitude is given, then an amplifier will convert low amplitude to high amplitude signals.
- Amplifier is *n-p-n* transistor with common emitter configuration.
- An amplifier have two input and two output terminals where a transistor as an amplifier have one of its three terminals common to both input and output.



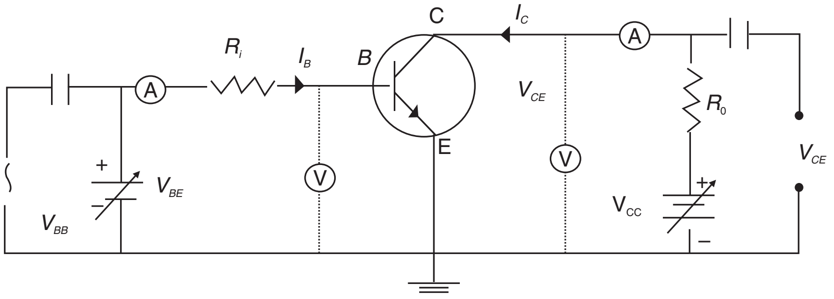


Circuit diagram of transistor as an amplifier

Common Emitter Mode

- The common function of a transistor is to be used in Common Emitter mode where small changes in base/emitter current results in large changes in collector/emitter current.
- In a current amplifier, for having voltage amplification, load resistor is connected in collector circuit, so that a change in collector current results in change in the voltage that is developed across the load resistor.
- The value of load resistor will affect the voltage gain of an amplifier.
- In an amplifier, larger is the load resistor, larger will be the change in voltage which results from change in collector current.
- In this, output waveform is in anti-phase to input waveform that takes place due to increase in base/emitter voltage which increases the base current.
- Due to an increase in collector current, voltage drop across the load resistor also increase.
- As voltage on the top end of the load resistor remains same, voltage on the bottom end tends to decrease which increases the base/emitter voltage while decreases the collector/emitter voltage.

- In transistor *C-E* amplifier with *n-p-n* transistor, a transistor to work as an amplifier should be forward biased with *C-B* junction to be reversed biased.



In the circuit :

- voltage sources V_{BB} and V_{CC} gives required bias.
- signal voltage V_i to be amplified is applied between the base and emitter with development of amplified voltage V_0 across the load resistor R_0.

- **For the common emitter transistor amplifiers :**

(i) Current gain : It is of two types :

(a) ***d.c***. gain **(β_{dc}) :** It is defined as the ratio of the collector current (I_C) to the base current (I_B). That is

$$\beta_{DC} = \frac{I_C}{I_B}$$

(b) a.c. gain, (β_{ac}) : It is defined as the ratio of the change in collector current (ΔI_C) to the change in base current (ΔI_B) at constant collector emitter voltage (V_{CE}). That is :

$$\beta_{ac} = \left[\frac{\Delta I_C}{\Delta I_B}\right]_{V\,\text{CE–constant}}$$

(ii) Voltage gain (A_V) is defined as the ratio of the change in output voltage (ΔV_0) to that of the input voltage (ΔV_i).

i.e., $$A_V = \frac{\Delta V_0}{\Delta V_i}$$

Here V_o = collector emitter voltage and V_i = base emitter voltage.

(iii) Resistance gain (A_R) is defined as the ratio of output resistance (R_0) to the input resistance (R_i).

i.e., $$A_R = \frac{R_0}{R_i}.$$

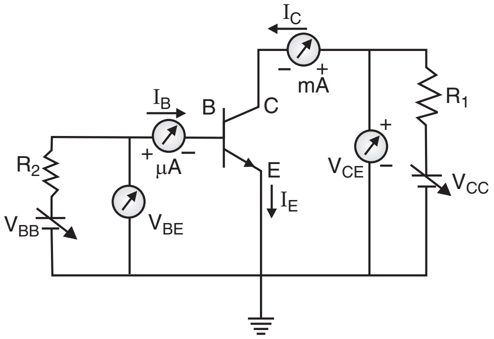
(iv) Power gain : A_P is defined as the ratio of the change in output power (DP_0) to that in the input power (DP_i).

i.e., $$A_P = \frac{\Delta P_0}{\Delta P_i}.$$

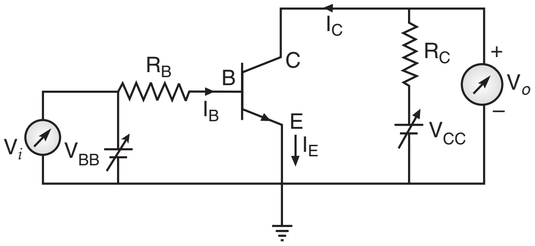
Some Important Circuit Diagrams and Graphs

(A) Circuit Diagrams

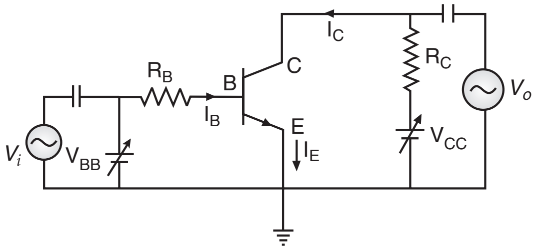
(a) Circuit arrangement for studying the input and output characteristics of NPN transistor in CE configuration.



(b) Base-biased transistor in CE configuration.



(c) A simple circuit of a CE-transistor amplifier.



Know the Formulae

(A) $I_C = \beta I_B$

For the common emitter transistor amplifiers :

- ➢ $\beta_{DC} = \dfrac{I_C}{I_B}$
- ➢ $\beta_{ac} = \left[\dfrac{\Delta I_C}{\Delta I_B}\right]_{V_{CE\text{-constant}}}$
- ➢ $A_V = \dfrac{\Delta V_o}{\Delta V_i}$
- ➢ $A_R = \dfrac{R_o}{R_i}$
- ➢ $A_P = \dfrac{\Delta P_o}{\Delta P_i}$

For the common base transistor amplifiers :

- ➢ $\alpha_{DC} = \dfrac{I_C}{I_E}$
- ➢ $\alpha_{ac} = \left[\dfrac{\Delta I_C}{\Delta I_E}\right]_{V_{CB}=\text{constant}}$
- ➢ $A_V = \dfrac{\Delta V_o}{\Delta V_i}$
- ➢ $A_R = \dfrac{R_o}{R_i}$
- ➢ $A_P = \dfrac{\Delta P_o}{\Delta P_i}$
- ➢ $a = \dfrac{\beta}{1+\beta}$ and $\beta = \dfrac{\alpha}{1-\alpha}$

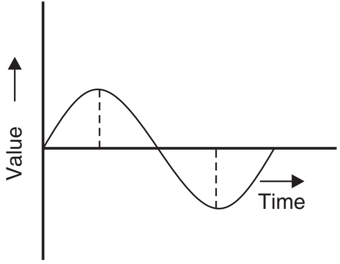
TOPIC-4
Logic Gates

Quick Review

Analog signal

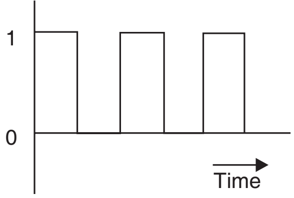
- ➢ Analog signal is continuous wave or signal for which the time varying feature of the signal is representation of time varying quantity.
- ➢ It is classified as simple signal and composite signal.
- ➢ Simple analog signal is a sine wave which cannot be decomposed further while composite analog signal can be further decomposed in multiple sine waves.
- ➢ Analog signal can be described by amplitude, period or frequency and phase where amplitude shows maximum height of the signal, frequency shows rate at which signal is changing while phase shows the position of the wave with respect to time zero.

- Analog signal is different from digital signal in terms of small fluctuations in signal.



Digital signal

- Digital signal make use of discontinuous values and carry information like analog signals.
- The signal carries information or data in binary form as bits and make use of numbers, letters, sounds, images and other measurements of continuous systems.
- The signal can be decomposed into simple sine waves known as harmonics having different amplitude, frequency and phase.
- Such signal can be described with bit rate and bit interval where bit interval shows time require for sending a single bit while bit rate shows the frequency of bit interval.



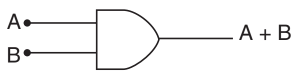
Comparison of Analog and Digital Signals

Analog signal	Digital signal
It is a continuous signal which represents physical measurements.	It is a discrete time signal generated by digital modulation.
It is shown by sine waves.	It is shown by square waves.
It make use of continuous range of values to describe an information.	It make use of discrete or discontinuous values to describe an information.
The examples of this are human voice in air, analog electronic devices.	The example of this are computers, CDs, DVDs, and other digital electronic devices.
It records waveforms as they are.	It samples analog waveforms in limited set of numbers and records them.
It is subjected to deterioration by noise at the time of transmission and write/read cycle.	It is noise-immune without deterioration at the time of transmission and write/read cycle.
It is described by amplitude, period, frequency and phase.	It is described by bit rate and bit intervals.
It is used in analog devices only.	It is used both in analog and digital devices
Processing of signal is done in real time and consumes less bandwidth.	Digital signal has no guarantee of processing in real time and consumes more bandwidth.
It is stored in form of wave signal.	It is stored in form of binary bit.
In this, analog instrument draws large power.	In this, digital instrument draws negligible power.
It has low cost and are portable.	It has high cost and are not easily portable.
It transmit data in form of wave.	It carries data in binary form as 0 and 1.
It involves in cramped at lower end and give considerable observational errors.	It is free from observational errors such as parallax and approximation errors.

Logic gates (OR, AND, NOT, NAND and NOR)

- Digital systems are constructed using basic logic gates.
- Gate is an electronic circuit that works with one or more than one signals to generate output signal.
- Digital circuits or logic circuits has input and output signals that are either low (logic 0) or high (logic 1).
- Basic logic gates are; AND, OR, NOT, NAND and NOR.
- Truth table shows all possible values of logical variables/statements along with all the possible results of given combinations of logic gates values.

OR gates

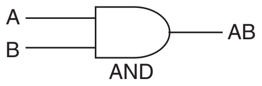


2 Input OR Gate		
A	**B**	**A+B**
0	0	0
0	1	1
1	0	1
1	1	1

The OR gate circuit shown has two inputs and single output.
The output of OR gate is high (1) if one or more than one of its inputs are also high.
The output of OR gate shown with plus (+) shows the total of two inputs.

AND gates

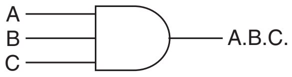
- In an AND gate, output is high (1) if all the inputs are high.
- In this, dot (.) shows the AND operation *i.e.*, A.B.
- In case of two input and one output, the truth table of AND gate can be written as shown.



2 Input AND Gate		
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

Three Input AND Gate

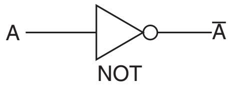
- In three input AND gate, truth table is similar with two input gate.
- Three input AND gate works on the principle where all three inputs need to be high (1) to have a high output.



3 Input AND Gat			
A	*B*	*C*	*ABC*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

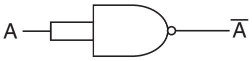
NOT gates

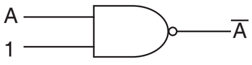
- In a NOT gate circuit with single input and single output, the output is inverted of input.
- Due to inverted output, NOT gate circuit is known as an inverter circuit.



NOT gate	
A	*A*
0	1
1	0

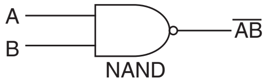
- In this, when input variable is A, the output will be an inverted variable of A known as NOT A.
- The output will be A′ or Â as shown.





- NOT gate can be obtained by NAND gate or NOR gate.

NAND gates

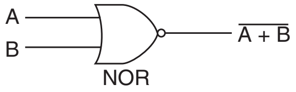


2 Input NAND Gate		
A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

- NAND gate is called as NOT-AND gate similar as AND gate followed by NOT gate.
- In NAND gate, the output is high if any of its inputs are low.
- NAND gate is show by AND gate with circle on output.
- Due to the small circle at the output, output is inverted value of input.

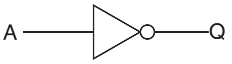
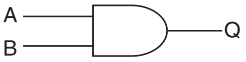
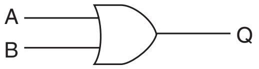
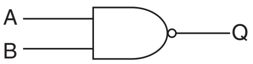
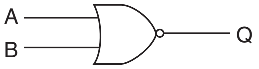
NOR gates

- NOR gate is called as NOT-OR gate which is similar as OR gate along with NOT gate.
- In NOR gate circuit, output of the gate is low if any of its inputs are high.
- NOR gate is shown by OR gate with circle on the output.
- Due to the small circle at the output, output of NOR gate is inversion of its input.



2 Input NOR Gate		
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

By putting AND, OR, and NOT gates together, we can construct more complex (logic) functions :

Gate	Symbol	Boolean Notation	
NOT		$Q = \overline{A}$	(read as Q = NOT A)
AND		$Q = A.B$	(read as Q = A AND B)
OR		$Q = A + B$	(read as Q = A OR B)
NAND		$Q = \overline{A.B}$	(read as Q = A NAND B)
NOR		$Q = \overline{A+B}$	(read as Q = A NOR B)

Know the Terms

- **Logic Gates:** An elementary building block of digital circuit having two inputs and one output.
- **Analog signal:** A continuous wave or signal for which time varying feature of signal is representation of time varying quantity
- **Digital signal:** It is a discrete time signal generated by digital modulation

Know the Formulae

- NOT Operator—It operates on single variable. It gives the complement value of variable.

X	X'
0	1
1	0

- OR Operator—It is a binary operator and denotes a logical addition operation and is represented by symbol (+).

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

X	Y	$X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

- AND Operator—AND operator performs the logical multiplication and its symbol is (.) dot.

$0.0 = 0$
$0.1 = 0$
$1.0 = 0$
$1.1 = 1$

X	Y	$X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

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UNIT - X : Communication

Chapter - 15 : Communication Systems

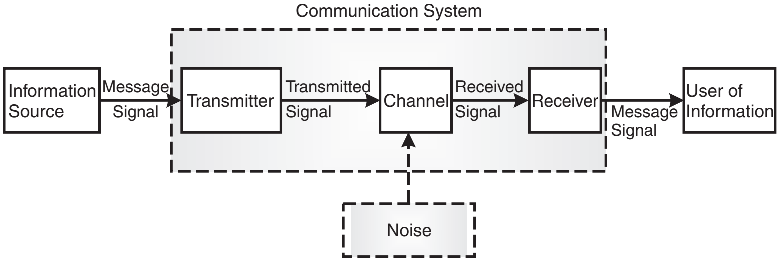
TOPIC-1

Communication System

Quick Review

- Communication is the method of transmitting and receiving information.
- There may be two types of communication :
  - **Point to point communication :** In this type of communication transmitter and receiver are single point. **Example :** Telephonic communication.
  - **Broadcasting communication :** In this type of communication, there is one transmitter and many receivers. **Example :** transmission in radio or television.
- When input message (any form of signal variation) is combined with some intelligence input (encoding and compatibility in machine language), it becomes information.

- **Generalised communication system :**



- **Important terminology and their application in communication system**
- **Information source :** The information source is the generator of information which we want to communicate. It may be audio, video or data.
- **Electric Transducer :** Electric transducer convert physical variable into electrical signal variable.
- **Signal:** Information converted in electrical form and suitable for transmission is called signal.
- **Transmitter :** Process the incoming message signal and making it suitable for transmission through particular channel.
- **Noise :** Unwanted signal which interferes with the information signal and disturbs the information.
- **Channel :** It is the medium through signal from transmitter propagates to the receiver. **For example :** optical fibre, coaxial cables etc.
- **Receiver :** Collect the message from the channel and extract signal.
- **Attenuation :** Signal loss energy during propagation through channel. This is called attenuation.
- **Amplifier :** It is the device which increase the strength by increasing its amplitude.
- **Range :** Maximum distance between transmitter and receiver at which signal can be recovered is called the range of communicating system.
- **Bandwidth :** It is the frequency range over which an equipment operates or range of frequencies a signal has.
- **Modulation :** Mixing of signal with carrier frequency is known as modulation.
- **Demodulation :** Extracting of signal from carrier frequency is known as demodulation.
- **Repeater :** It receives the signal, reconditions it and then retransmit it.

Signal may be classified in two categories :

- **Analog Signal :** Continuous variation of signal with respect to time is known as analog signal. **For example :** telephonic signal, video signal etc.

  Now with modern technology we can convert analog signal to digital signal for communicating and convert back to analog signal at receiver.
- **Digital Signal :** Discrete value of signal variation with respect to time is known as digital signal. **For example :** computer etc.
  - Coding helps in sending digital signal with much more accuracy. There are several coding techniques. Like in computer data, we employ suitable combinations of number systems such as the binary coded decimal (BCD), American Standard Code for Information Interchange (ASCII)
- **Operational advantages of digital communication system over analog communication systems are.**
  - An improved form of sending messages securely.
  - Increased immunity to noise and external interference.
  - A common format for encoding different kinds of message signals for the purpose of transmission.
  - Flexibility in configuration of digital communication system.
- Hence in modern technology, analog signals are transmitted through digital communication. In the final stage they are converted back to analog signals.
- Different types of message signals have different range of frequencies.
  - Audio signal – 20 Hz to 20kHz
  - Video signal – 4.2 MHz
  - TV signal – 6 MHz (audio + video)
- Large bandwidth is required to accommodate complete information of wave.
- **Frequency bands of some important wireless communications :**

Service	Frequency bands	Comments
Standard AM broadcast	540-1600 kHz	
FM broadcast	88-108 MHz	
Television	54-72 MHz	VHF (very high frequencies TV)
	76-88 MHz	TV

	174-216 MHz	UHF (ultra high frequencies)
	420-890 MHz	TV
Cellular Mobile Radio	896-901 MHz	Mobile to base station
	840-935 MHz	Base station to mobile
Satellite Communication	5.925-6.425 GHz	Uplink
	3.7-4.2 GHz	Downlink

Propagation of Electromagnetic wave :

- Earth's atmosphere plays a vital role in propagation of electromagnetic wave. There are three ways of communication through electromagnetic wave.
- **Ground Wave :**
  - **(i)** The radio waves which travel through atmosphere following the surface of the earth are called ground waves or surface waves and their propagation is known as ground wave propagation or surface wave propagation.
  - **(ii)** The ground waves have vertical orientation and travel parallel to the ground.
  - **(iii)** The ground wave propagation is suitable for low and medium frequency, *i.e.*, from few hundred kHz to 2 MHz only.
  - **(iv)** Its power is less as they operate in low frequency.
  - **(v)** It can bend round the corners of the object on the earth, hence can jump the restriction. ($\theta = \frac{\lambda}{a}$, low frequency means λ is more so more bending)
  - **(vi)** Attenuation is high for ground wave transmission and increases with increase in frequency. This is because more absorption of ground waves (near earth) takes place at higher frequency during propagation through atmosphere.
  - **(vii)** Length of antenna is directly proportional to the wavelength of *EM* wave. Hence, for ground wave large antenna is required.
  - **(viii)** The ground wave propagation is generally used for local band broadcasting and is commonly known as medium wave. Local transmitter, police walkie talkie , AM transmitter are some of its examples.

Sky wave propagation :

- **(i)** The sky waves are the radio waves of frequency between 2 MHz to 30 MHz.
- **(ii)** These radio waves can propagate in atmosphere and are reflected back by the ionosphere of earth's atmosphere.
- **(iii)** The sky waves can go from transmitter antenna to receiver antenna, while travelling through sky and after reflection from ionosphere. Hence, their propagation is called sky wave propagation.
- **(iv)** Critical frequency (f_c) is that highest frequency of radio waves, which when sent straight (*i.e.*, normally) towards the layer of ionosphere gets reflected and returns to the earth. If the frequency of radio waves is more than the critical frequency, it will not be reflected by the ionosphere.
- **(v)** The value of *C.F.* is found to be 4 MHz, 5 MHz and 6 to 8 MHz for *D* (part of stratosphere), *E* (part of stratosphere), F_1 (part of mesosphere) and F_2 (Thernhosphere) layers of ionosphere which are at heights about 110 km, 180 km and 300 to 350 km respectively from the surface of earth.
- **(vi)** Its range is very large as compared to range of ground wave. Range can be targeted and can be increased by multiple transmitters.
- **Limitations :** 3 MHz to 30 MHz is very small bandwidth of frequency for present application. Higher frequencies penetrate the ionosphere and can't be reflected.

Space wave propagation :

- **(i)** It is used for very high frequency (> 40 MHz). These can penetrate ionosphere more efficiently.
- **(ii)** Due to high frequency, wavelength is very small and energy is very high.
- **(iii)** Television broadcast, microwave links and satellite communication are some examples of communication systems that use space wave mode of propagation.

Line of sight communication by space wave :

- We also use this space wave in ground transmission. It is known as line of sight transmission.
- These are (space wave) high frequency hence they travel nearly in a line. Mobile transmission or microwave links are based upon this.
- Earth's curvature restrict the range of line of sight transmission. There is limited space between two antennas.
- If h is the height of transmitting antenna then its signal range is $d = \sqrt{2hR}$
- The range of communication d_M between the transmitting antenna of height h_T and the receiving antenna of height h_R is given by $d_M = \sqrt{2h_TR} + \sqrt{2h_RR}$

  where, R is the radius of the earth.

$$\text{Area covered through one tower} = \pi d^2 = \pi \times 2hR$$

$$\text{Population covered} = \text{population density} \times \text{area covered.}$$

Satellite Communication :

- The satellite communication is a mode of communication of signal between a transmitter and a receiver through a satellite.
- The satellite communication is like a line of sight microwave communication.
- Since, the satellite communication is through space hence, it is also part of space communication.
- A communication satellite is a space craft, provided with microwave receiver and transmitter. It is placed in an orbit around the earth.
- In satellite communication, a beam of modulated microwave from the transmitter is sent directly towards the communication satellite, which receives the coming signal, amplifies it and returns it to the earth. Transmitting frequency (uplink) and receiving frequency (downlink) are different to avoid interference between the uplink and the downlink.
- A satellite communication is possible through geostationary satellites.
- A single geostationary satellite cannot cover the whole part of the earth for microwave communication. It is so because, the large part of the earth is out of sight due to the curvature of the earth. One satellite roughly covers one third of earth.
- In order to have global transmission, at least three geostationary satellites are required, which are at particular distance from each other.
- Global positioning system is also based upon satellite communication.

TOPIC-2

Modulation

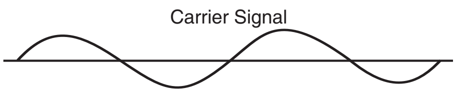
Quick Review

Low frequency signal could not travel large distance because of following reasons :

- Low frequency means low power, hence it gets attenuated i.e., loss of signal strength.
- Size of antenna = $\frac{\lambda}{4}$ and low frequency means large wavelength so size of antenna becomes impractical.
- Overlapping of signal. Difficult to incorporate multiple transmitting stations.
- Hence the signal should be transmitted at high frequency.
- Combining low frequency message signal with high frequency carrier wave is modulation.
- A high frequency wave has certain features like amplitude, frequency and phase.

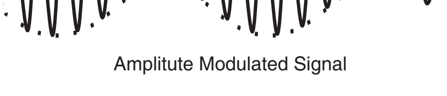
  $$y = a\cos(\omega t + \phi)$$

  So, variable parameters are amplitude (a), frequency ν and phase (ϕ).
- Depending upon the parameter which we are varying in carrier wave with our signal, there are three main types of modulation techniques.
  - Amplitude modulation
  - Frequency modulation
  - Phase modulation
- **Amplitude Modulation :** The amplitude of the carrier wave changes according to the intensity of the signal. The amplitude variation of the carrier wave is at the signal frequency f_s.



Carrier Signal

Modulating Sine Wave Signal



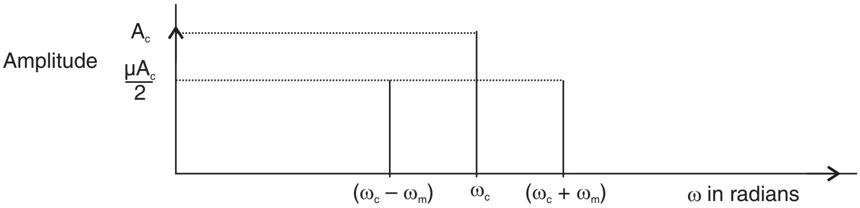
Amplitute Modulated Signal

If message signal $m(t) = A_m \sin \omega_m t$

carrier wave $c(t) = A_c \sin \omega_c t$

are combined together then bandwidth of modulated wave is $(\omega_c - \omega_m)$ to $(\omega_c + \omega_m)$ *i.e.*, $2\omega_m$.

- $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$ are known as lower and upper sideband frequency respectively. Signal is in these side band frequencies.



Modulation Factor : The ratio of change of amplitude of modulated wave to the amplitude of normal carrier wave is called modulation index (μ).

$\mu = \frac{A_m}{A_c}$ To prevent distortion $\mu \leq 1$.

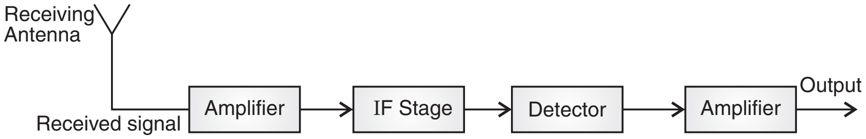
We can derive that $A_m = \frac{A_{max} - A_{min}}{2}$ and $A_c = \frac{A_{max} + A_{min}}{2}$

Hence, $\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

- **Effect of Noise on AM wave :** AM signal is more noisy than *FM* because in *AM* message is transmitted through modulating the amplitude of carrier signal. A low frequency noise can alter the amplitude of carrier message. In frequency modulation message is transmitted through frequency changes and hence amplitude of noise signal will not effected.

Detection of amplitude modulated wave :

- **Demodulation :** Demodulation is the process of recovering the signal frequency from a modulated carrier wave.
- The detected signal may not be strong enough to be made use of and hence is required to be amplified.
- Below is the block diagram of typical receiver circuit.



- **Other communicating modes**

  Internet

  Facsimile (FAX)

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