MIND MAP : LEARNING MADE SIMPLE

CHAPTER - 1

SETS

Subset

Let A and B be two sets. If every element of A is an element of B,then A is called a subset of B and written as $A \subset B$ or $B \supset A$(read as 'A' is contained in 'B' or 'B contains A'). B is called superset of A.

Note:

1. Every set is a subset and superset of itself.
2. If A is not a subset of B, we write $A \not\subset B$.
3. The empty set is the subset of every set.
4. If A is a set with n(A) = m, then no. of element A are 2^n and the number of proper subsets of A are 2^n-1

Eg. Let A = {3, 4}, then subsets of A are ϕ, {3}, {4}, {3, 4}. Here, n(A) = 2 and number of subsets of A = 2^2=4.

Cardinal Number

The number of elements in a finite set is represented by n (A), known as cardinal number.

Eg.: A = {a, b, c, d, e} Then, n (A) = 5

Introduction

A set is collection of well-defined distinguished objects. The sets are usually denoted by capital letters A, B, C etc., and the members or elements of the set are denoted by lower case letters a, b, c etc.., If x is a member of the set A, we write $x \in A$ (read as 'x' belongs to A) and if x is not a member of set A, we write $x \notin A$(read as 'x' doesn't belongs to A). If x and y both belong to A, we write $x, y \in A$.

Some examples of sets are: A: odd numbers less than 10
N: the set of all rational numbers
B : the vowels in the English alphabates
Q: the set of all rational numbers.

Representation of Sets

Set builder form or Rule Method

In this form, we write a variable (say x) representing any member of the set followed by property satisfied by each member of the set.

eg.: The set A of all prime number less than 10 in set builder form is written as

A ={x | x is a prime number less than10}

The symbol "|" stands for the word "such that". Sometimes, we use symbol ":" in place of symbol "|"

Roaster or Tabular form

In this form, we first list all the members of the set within braces (curly brackets) and separate these by commas.

Eg: The set of all natural number less than 10 in this form is written as: A = {1, 3, 5, 7, 9}

In roaster form, every element of the set is listed only once.

The order in which the elements are listed is immaterial.

Eg. Each of the following sets denotes the same set {1, 2, 3}, {3, 2, 1}, {1, 3, 2}

Types of Sets

Empty set or Null set

A set which has no element is called null set. It is denoted by symbol ϕ or {}.

E.g: Set of all real numbers whose square is −1.

In set-builder form: {x:x is a real number whose square is −1}

In roaster form: { } or ϕ

Finite and Infinite set

A set which has finite number of elements is called a finite set. Otherwise, it is called an infinite set.

E.g.: The set of all days in a week is a finite set whereas the set of all integers, denoted by {……−2, −1, 0, 1, 2,……} or {x | x is an integer} is an infinite set.

An empty set ϕ which has no element is a finite set A is called empty or void or null set.

Equal set

Two sets A and B are set to be equal, written as A=B, if every element of A is in B and every element of B is in A.

e.g.: (i) A = {1, 2, 3,4} and B = {3, 1, 4, 2}, then A =B

(ii) A = {x: x−5 =0} and B = {x: x is an integral positive root of the equation $x^2 - 2x - 15 = 0$}

Then A= B

Singleton set

A set having one element is called singleton set.

e.g.: (i) {0} is a singleton set, whose only member is 0.

(ii) A = {x: 1<x <3, x is a natural number} is a singleton set which has only one member which is 2.

Equivalent set

Two finite sets A and B are said to be equivalent, if n(A) = n(B). Clearly, equal set are equivalent but equivalent set need not to be equal.

e.g.: The sets A = {4, 5, 3, 2} and B = {1, 6, 8, 9} are equivalent, but are not equal.

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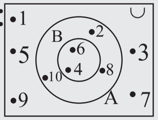
SETS

Venn Diagram

A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something common. These diagrams consist of rectangle and closed curves usually circles

Eg:



In the given venn diagram $U=\{1,2,3,.......10\}$ universe set of which $A=\{2,4,6,8,10\}$ and $B=\{4,6\}$ are subsets and also $B \subset A$

Algebra of sets

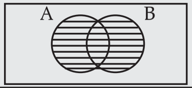
1. For any set A, we have
   (a) $A\cup A=A$, (b) $A\cap A=A$, (c) $A\cup\phi=A$, (d) $A\cap\phi=\phi$, (e) $A\cup U=U$
   (f) $A\cap U=A$, (g) $A-\phi=A$, (h) $A-A=\phi$
2. For any two sets A and B we have
   (a) $A\cup B = B\cup A$, (b) $A\cap B= B\cap A$, (c) $A-B\subseteq A$, (d) $B-A\subseteq B$
3. For any three sets A,B and C, we have
   (a) $A\cup(B\cup C)= (A\cup B)\cup C$, (b) $A\cap(B\cap C)= (A\cap B)\cap C$
   (c) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$, (d) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$
   (e) $A-(B\cup C)=(A-B)\cap(A-C)$, (f) $A-(B\cap C)=(A-B)\cup(A-C)$

Operations on Sets

Union

The union of two sets A and B, written as $A\cup B$ (read as A union B) is the set of all elements which are either in A or in B in both.
Thus, $A\cup B = \{x: x \in A \text{ or } x \in B\}$
clearly, $x \in A\cup B \Rightarrow x \in A$ or $x \in B$ and $x \notin A\cup B \Rightarrow x \notin A$ and $x \notin B$

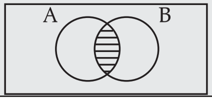
$A\cup B$



eg: If $A = \{a, b, c, d\}$ and $B =\{c, d, e,f\}$ then $A\cup B = \{a, b, c, d, e, f\}$

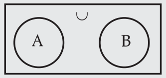
Intersection

The intersection of two sets A and B, written as $A\cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B.
Thus, $A\cap B = \{x : x\in A \text{ and } x\in B\}$
Clearly, $x\in A\cap B \Rightarrow \{x\in A \text{ and } x\in B\}$ and $x\notin A\cap B \Rightarrow \{x \notin A \text{ or } x \notin B\}$.
Eg: If $A=\{a,b,c,d\}$ and $B=\{c,d,e,f\}$
Then $A\cap B = \{c,d\}$



Disjoint sets

Two sets A and B are said to be disjoint, if $A\cap B =\phi$ i.e, A and B have no common element. e.g: if $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$
Then, $A\cap B =\phi$, so A and B are disjoint.

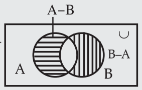


Symmetric Difference

The symmetric difference of two sets A and B, denoted by $A \Delta B$, in defined as $(A \Delta B)=(A-B) \cup (B-A)$
Eg. If $A = \{1, 2,3,4,5\}$
and $B = \{1,3,5,7,9\}$ then
$(A \Delta B)= (A-B) \cup (B-A)$
$= \{2, 4,\} \cup \{7, 9\}$
$= \{2, 4, 7, 9\}$

Difference of two sets

If A and B are two sets, then their difference A-B is defined as:
$A-B = \{x : x \in A \text{ and } x \notin B\}$
Similarly, $B-A = \{x : x \in B \text{ and } x \notin A\}$
Eg. If $A=\{1,2,3,4,5,\}$ and $B=\{1,3,5,7,9\}$
then
$A-B=\{2,4\}$ and $B-A=\{7,9\}$



Subsets of a set of real numbers 'R'

- The set of natural numbers
  $N=\{1, 2, 3, 4, 5, ---\}$
- The set of integers
  $Z=\{...-3, -2, -1, 0, 1, 2, 3,---\}$
- The set of irrational numbers,
  $T = \{x: x\in R \text{ and } x\in Q\}$
- The set of rational number
  $Q=\{x : x =\frac{p}{q}, p, q \in Z \text{ and } q \neq 0\}$
  Relation among these subsets are
  $N \subset Z \subset Q, Q \subset R, T \subset R, N \not\subset T$

Interval Notation

Let a and b be real numbers with a < b

Set of Real Numbers	Interval Notation	Region on the real number line
• $\{x \mid a< x < b\}$	(a, b)	○——○
• $\{x \mid a \le x < b\}$	[a, b)	●——○
• $\{x \mid a< x \le b\}$	(a, b]	○——●
• $\{x \mid a \le x \le b\}$	[a, b]	●——●
• $\{x \mid x < b\}$	$(-\infty, b)$	←——○
• $\{x \mid x \le b\}$	$(-\infty, b]$	←——●
• $\{x \mid x >a\}$	(a, ∞)	○——→
• $\{x \mid x >a\}$	$[a, \infty)$	●——→
• R	$(-\infty, \infty)$	←——→

Universal Set

The set containing all objects of element and of which all other sets are subsets is known as **universal sets** and denoted by U.
E.g : For the set of all intergers, the universal set can be the set of rational numbers or the set R of real numbers

Power Set

The set of all subset of a given set A is called **power set** of A and denoted by P(A).
E.g : If $A =\{1,2,3,\}$, then $P(A)=\{\phi\}, \{1\},\{2\}, \{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}$.
Clearly, if A has n elements, then its power set P (A) contains exactly 2^n elements.

Compliment of Set

If U is a universal set and A is a subset of U, then complement of A is the set which contains those elements of U, which are not present in A and is denoted by A′ or A^C. Thus,
$A^C = \{x: x\in U \text{ and } x\notin A\}$
e.g.: If $U = \{1, 2, 3, 4, ...\}$
and $A = \{2, 4, 6, 8, ...\}$
then $A^C = \{1, 3, 5, 7, ...\}$

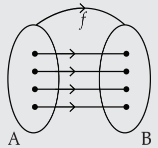
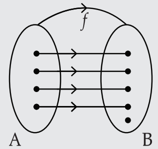
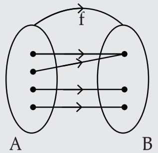
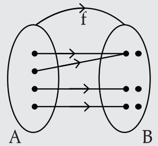
Properties of complement

- Complement law:
  (i) $A\cup A' = U$ (ii) $A\cap A'= \phi$
- De morgan's Law:
  (i) $(A\cup B)' = A'\cap B'$ (ii) $(A\cap B)' = A'\cup B'$
- Double Complement law:
  $(A')' =A$
- Law of empty set and universal set
  $\phi'=U$ and $U'=\phi$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 2

Relations & Functions

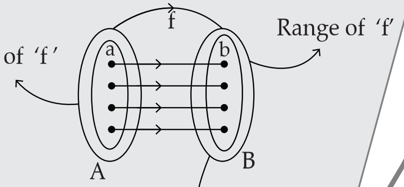
Kinds of Functions

One-One Onto function	One-One Into function
 Range=Codomain	 Range ⊂ Codomain
Many-One Onto function	**Many One Into function**
 Range=Codomain	 Range=Codomain

Functions

Definition: A relation ' f ' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Notations: Domain(Input) $\xrightarrow{x}$ [f] ⟶ y=f(x) Range(Output)



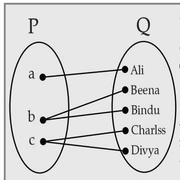
Cartesian product of sets

Given two non empty sets A&B. The cartesian product A×B is the set of all ordered pairs of elements from A&B i.e., A×B={(a,b) : a ∈ A ; b ∈ B}. If n(A)=p and n(B)=q , then n(A×B)=pq

Relation

Let A&B be two empty sets. Then any subset ' R ' of A x B is a relation from A to B.If (a,b) ∈R, then we write a R b , which is read as ' a is related to b ' by a relation R, 'b' is also called image of'a' under R. The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A x B. If n(A)=p and n(B)=q, then n (A x B)=pq and total number relations is 2^{pq}.

Pictorial representation of a relation



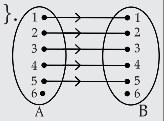
Given, R={(x,y): x is the first letter of the name y, x ∈P, y ∈Q} Then, R={(a,Ali), (b, Beena), (c, charu)}This is a visual or pictorial representation of relation R (called an arrow diagram)is shown in figure.

Domain & range of a Relation

If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. Symbolically, Domain of R={x:(x,y) ∈ R}; Range of R={y:(x,y) ∈ R} The set B is called co-domain of relation R.

Note: the range ⊆ Codomain.

Eg. Given, R={(1,2),(2,3),(3,4),(4,5),(5,6)}. then Domain of R={1,2,3,4,5} Range of R={2,3,4,5,6} and codomain of R={1,2,3,4,5,6}



Inverse relation

Let A&B be two sets and R be a relation from set A to set B.Then inverse of R ,denoted by R^{-1}, is a relation from B to Aand is defined by R^{-1}={(b,a) : (a,b) ∈ R}. Clearly, (a,b) ∈R ⇔ (b,a) ∈ R^{-1} Also, Dom(R)=Range (R^{-1}) and Range(R)=Dom(R^{-1})

Even and odd function

Even function
$f(-x)=f(x)$, ∀x∈ Domain

Odd function
$f(-x)=-f(x)$, ∀x∈ Domain

Algebra of functions

Let $f : x \to R$ and $g \to R$ be any two real functions where X⊂R.
Addition: (f+g) x = f(x)+g(x); ∀x∈ R
Subtraction: (f–g) x = f(x)–g(x); ∀ x∈ R
Product: (fg) x = f(x).g(x); ∀ x ∈ R
Quotient: (f/g) (x) = f(x)/g(x); provided g(x) ≠ 0, ∀ x ∈ R

Some standard real functions

Log function: $f(x)=\log_e x$, a>0 , a ≠ 1 Domain=x ∈ (0, ∞) Range=y ∈ R

Identity function: The function f:R → R defined by y=f(x)=x ∀ x ∈R is called identity function. Domain=R and Range=R

Constant function: The function f:R → R defined by y=f(x)=c, ∀ x ∈ R, where c is a constant is called constant function. Domain=R and Range ={c}

Modulus function: The function f:R → R defined by $f(x)=\begin{cases} x; x \ge 0 \\ -x; x < 0 \end{cases}$ is called modulus function. It is denoted by y=f(x)=|x|. Domain=R and Range =(0, ∞)

Signum function: The function f:R → R defined by $f(x)=\begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$ is called signum function. It is usually denoted by y=f(x)=sgn(x) Domain=R and Range ={ 0, -1, 1}

Greatest integer function: The function f:R → R defined by as the greatest integer less than or equal to x .It is usually denoted by y=f(x)=[x]. Domain=R and Range=Z(All integers)

Exponential function: $f(x)=a^x$, a>0 , a ≠ 1, Domain: x∈R: Range:f(x) ⊂ (0,∞)

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 3

Trigonometric Functions

Important Trigonometric Ratios values

- $\sin 15°$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$ or $\cos\frac{5\pi}{12}$
- $\cos 15°$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$ or $\sin\frac{5\pi}{12}$
- $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75°$
- $\tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15°$
- $\sin\frac{\pi}{10}$ or $\sin 18° = \frac{\sqrt{5}-1}{4}$
- $\cos\frac{\pi}{5}$ or $\cos 36° = \frac{\sqrt{5}+1}{4}$

Factorisation of the Sum or Difference of sine and cosine with two variables

- $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

Transformation of Product into Sum or Difference of Sines or Cosines

- $2\sin A \sin B = \sin(A+B) + \sin(A-B)$
- $2\cos A \sin B = \sin(A+B) - \sin(A-B)$
- $2\cos A \cos B = \cos(A+B) + \cos(A-B)$
- $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

Trigonometrical Function sum and Difference of Two Angles

- $\sin(A\pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A\pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A\pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\cot(A\pm B) = \frac{\cot A \cot B \mp 1}{\cot A \pm \cot B}$
- $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$
- $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B)\cos(A-B)$

Degree & Radian Measures

If in a circle of radius r, an arc of length l subtends an angle of θ radians , then $l = r\theta$.

- Radian Measure $= \frac{\pi}{180} \times$ Degree Measure
- Degree Measure $= \frac{180}{\pi} \times$ Radian Measure

Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1, -1 \le \sin\theta \le 1, -1 \le \cos\theta \le 1\ \forall\theta\in R$
- $\sec^2\theta - \tan^2\theta = 1, |\sec\theta| \ge 1, \forall\theta\in R$
- $\text{cosec}^2\theta - \cot^2\theta = 1, |\text{cosec}\,\theta| \ge 1, \theta\in R$

Trigonometrical Ratio of allied Angles

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\cot(-\theta) = -\cot\theta$
- $\sec(-\theta) = \sec\theta$
- $\text{cosec}(-\theta) = -\text{cosec}\theta$

- $\sin\theta\left(\frac{\pi}{2}\mp\theta\right) = \cos\theta$
- $\cos\left(\frac{\pi}{2}\mp\theta\right) = \pm\sin\theta$
- $\tan\left(\frac{\pi}{2}\mp\theta\right) = \pm\cot\theta$
- $\cot\left(\frac{\pi}{2}\mp\theta\right) = \pm\tan\theta$
- $\sec\theta\left(\frac{\pi}{2}\mp\theta\right) = \pm\text{cosec}\theta$
- $\text{cosec}\left(\frac{\pi}{2}\mp\theta\right) = \sec\theta$

- $\cos((\pi\mp\theta) = -\cos\theta$
- $\sin(\pi\mp\theta) = \pm\sin\theta$
- $\tan(\pi\mp\theta) = \mp\tan\theta$
- $\cot(\pi\mp\theta) = \mp\cot\theta$
- $\sec(\pi\mp\theta) = -\sec\theta$
- $\text{cosec}(\pi\mp\theta) = \pm\text{cosec}\theta$

- $\sin(2\pi\mp\theta) = \mp\sin\theta$
- $\cos(2\pi\mp\theta) = \cos\theta$
- $\tan(2\pi\mp\theta) = \mp\tan\theta$
- $\cot(2\pi\mp\theta) = \mp\cot\theta$
- $\sec(2\pi\mp\theta) = \sec\theta$
- $\text{cosec}(2\pi\mp\theta) = \mp\text{cosec}\theta$

- $\sin\left(\frac{3\pi}{2}\mp\theta\right) = -\cos\theta$
- $\cos\left(\frac{3\pi}{2}\mp\theta\right) = \mp\sin\theta$
- $\tan\left(\frac{3\pi}{2}\mp\theta\right) = \pm\cot\theta$
- $\cot\left(\frac{3\pi}{2}\mp\theta\right) = \pm\tan\theta$
- $\sec\left(\frac{3\pi}{2}\mp\theta\right) = \mp\text{cosec}\theta$
- $\text{cosec}\left(\frac{3\pi}{2}\mp\theta\right) = \sec\theta$

Multiple and Half Angles

- $\sin 2A = 2\sin A\cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
  $= 2\cos^2 A - 1$
  $= 1 - 2\sin^2 A$
- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
- $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$
- $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$
- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$
- $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
- $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$
- $\tan\theta = 2\tan\frac{\theta}{2} / \left(1-\tan^2\frac{\theta}{2}\right)$

Trigonometrical Equations

The equation involving trigonometric functions of unknown angles are known as Trigonometric equations. e.g $\cos\theta = 0$, $\cos^2\theta - 4\cos\theta = 1$
A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. $\sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots$

Standard General Solutions of Trigonometrical ratios

<u>General solution:</u> The solution consisting of all possible solutions of a trigonometric equation is called its General Solution.

- $\sin\theta = 0 \Leftrightarrow \theta = n\pi$, $\cos\theta = 0 \Leftrightarrow \theta = (2n + \frac{\pi}{2})$
- $\tan\theta = 0 \Leftrightarrow \theta = n\pi$, $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\alpha$, where $\alpha \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
- $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi \pm \alpha$, where $\alpha \in [0, \pi]$
- $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$, where $\alpha \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
- $\sin^2\theta = \sin^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$, $\cos^2\theta = \cos^2\alpha \Leftrightarrow n\pi \pm \alpha$
- $\tan^2\theta = \tan^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$, $\sin\theta = 1 \Leftrightarrow \theta = (4n+1)\frac{\pi}{2}$
- $\cos\theta = 1 \Leftrightarrow \theta = 2n\pi$, $\cos = -1 \Leftrightarrow \theta = (2n+1)\pi$
- $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi + \alpha$, $\eta \in z$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 4

Principle of Mathematical Induction

Induction

Specific Instances to Generalisation
e.g.: Rohit eats food. Vikas eats food. Rohit and Vikas are men.Then all men eat food. Statement is true for n=1,n=k and n=k+1, then, the statement is true for all natural numbers n.

Deduction

Generalisation of Specific Instance
e.g.: Rohit is a man and all men eat food, therefore, Rohit eats food.

Steps for Principle of Mathematical Induction Proof

Step1: Let P(n) be a result or statement formulated in terms of n in a given equation
Step2: Prove that P(1) is true.
Step3: Assume that P(k) is true.
Step4: Using step 3 , prove that P(k+1) is true.
Step5: Thus, P(1) is true and P(k+1) is true whenever P(k) is true
Hence,by the principle of mathematical induction,P(n) is true for all natural numbers n.

Example

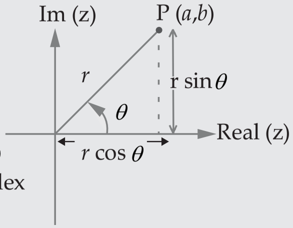
Ex: Prove that $2^n>n$ for all positive integer n.
Solution: Step1: Let P(1):$2^n>n$
Step2: When n=1, $2^1>1$.Hence P(1) is true
Step3: Assume that P(k) is true for any positive integer k, $2^k>k$
Step4: We shall now prove that P(k+1) is true
Multiplying both sides of step(3) by 2 , we get
$2.2^k>2\,k$
$\Rightarrow 2^{k+1}>2k$
$\Rightarrow 2^{k+1}= k+k$
$\Rightarrow 2^{k+1}> k+1$ (since k>1)
Therefore, P(k+1) is true when P(k)
is true Hence, by P.M.I., P(n) is true for every positive integer n .

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 5

Complex Number & Quadratic Equations

Polar Representation

Let $a = r\cos\theta$
$b = r\sin\theta$
where,
$r = |z|$
and $\theta = \arg(z)$
$\therefore z = a + ib = r(\cos\theta + i\sin\theta)$
The argument 'θ' of complex number z = a+ib is called principal argument of z if
$-\pi < \theta \le \pi$.

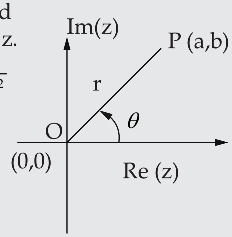


Modulus & Argument of Complex Number

If z= a+ib is a complex number
(i) Distance of z from origin is called as modulus of complex number z.

It is denoted by $r = |z| = \sqrt{a^2 + b^2}$

(ii) Angle θ made by OP with +ve direction of X-axis is called argument of z.



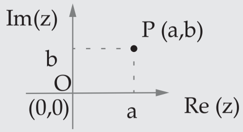
Powers of 'i'

$i = \sqrt{-1}, i^2 = -1$

In general, $i^{4k+r} = \begin{cases} 1; r = 0 \\ i; r = 1 \\ -1; r = 2 \\ -i; r = 3 \end{cases}$

Argand Plane

A complex number z=a+ib can be represented by a unique point P(a,b) in the argand plane



z=a+ib is represented by a point P (a,b)

Square root of Complex Number

Let $x + iy = \sqrt{a + ib}$, squaring both sides, we get $(x+iy)^2 = a+ib$ i.e. $x^2 - y^2 = a$, $2xy = b$ solving these equations, we get square root of z.

Multiplicative Inverse of complex Number

For a non-zero complex number z=a+ib $(a \neq 0, b \neq 0)$,

there exists a complex number $\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2} i$

denoted by $\frac{1}{z}$ or z^{-1}, called multiplicative inverse of Z

Such that: $(a+ib)\left(\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}\right) = 1 + 0i = 1$

Algebra of Complex Numbers

Let: $z_1 = a+ib$ and $z_1 = c+id$ be two complex numbers, where a,b,c,d ∈ R and $i = \sqrt{-1}$

1. Addition: $z_1 + z_2 = (a+ib) + (c+id) = (a+c) + i(b+d)$

2. Subtraction: $z_1 - z_2 = (a+ib) - (c+id) = (a-c) + (b-d)i$

3. Multiplication: $z_1 \cdot z_2 = (a+ib)(c+id)$
$= a(c+id) + ib(c+id)$
$= (ac - bd) + (ad + bc)i$
$(\because i^2 = -1)$

4. Division: $\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{a+ib}{c+id} \cdot \frac{c-id}{c-id}$
$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$

Note: If a+ib = c+id
⇔ a = c & b = d

Solution of Quadratic Equation

- General form of quadratic equation in x is $ax^2+bx+c=0$,
Where a,b,c ∈ R & a ≠ 0
The solutions of given quadratic equation are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$\because b^2-4ac<0$

Note:
- A polynomial equation has atleast one root.
- A polynomial equation of degree n has n roots.

Definition of Complex Numbers

A number of the form a+ib, where a,b ∈ R and $i = \sqrt{-1}$ is called a complex number and denoted by 'z'.

z= a+ib
a → Real part
ib → Imaginary part

Conjugate of a complex number: For a given complex number z=a+ib, its conjugate is defined as $\bar{z} = a - ib$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 6

Linear Inequalities

Notations

- The notation $a<b$ means that 'a is less than b'.
- The notation $a>b$ means that 'a is greater than b'.
- The notation $a\geq b$ means that 'a is greater than or equal to b'.
- The notation $a\leq b$ means that 'a is less than or equal to b'.
- The notation $a\neq b$ means that 'a is not equal to b'.

Solution of Linear Inequality in One Variable

Solution and Solution set

Solution: Values of x , which make inequality a true statement.

e.g.: '3' is the solution for $x<7$.

Solution Set:

The set of values of x is called its solution set.

e.g.: $\{1,2,3,4,5,6\}$ is solution set for $x<7$, where x is a natural number.

Types of Inequalities

Numerical Inequalities : Relation between numbers e.g.: $3<5$, $4>7$

Literal or Variable Ineqalities : Relation between two variables or variable and numbers. e.g.: $y<8$

Double Inequalities: Relationship from two sides e.g.: $2<x<5$

Strict Inequalities: An inequality that uses the symbol <or> e.g.: $x<5$, $3<5$

Slack Inequality: An inequality that uses the symbols $\leq$ or $\geq$. e.g.: $y\leq 5$

Linear Inequalities in One Variable: An inequality which involves a linear function in one variable e.g.: $x<5$

Linear Inequalities in two variables: An inequality which involves a linear function in two variables, e.g.: $3x+2y<5$

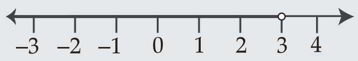
Quadratic Inequalities: An inequality which involves a quadratic function, eg.: $x^2+2x\leq 5$.

Representation

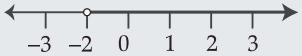
To represent $x<a$ ($x>a$) on a number line, put a circle on the number 'a' and dark line to the left (or right) of the number 'a'.

e.g:

$x<3$



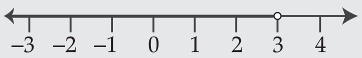
$x>-2$



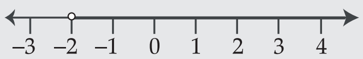
To represent $x\leq a$ (or $x\geq a$) on a number line, put a dark circle on the number 'a' and dark the line to the left (or right) of the number 'a'.

e.g:

$x\leq 3$



$x\geq -2$



Conditions

- An inequality is an relation that holds between two values, when they are different eg.: $x<5$, here, there is a relation between x & 5.
- Two real numbers or algebraic expressions related by symbol '<', '>', '≤' or '≥' form inequality.

eg,: $y\leq 4$, $2x+3y\geq 5$, $3>x$.

Rules of Inequality

- Equal numbers may be added to (or subtracted from) both the sides of an inequality without affecting the sign of inequality:
  e.g: $x<7$ is same as $x+2<7+2$
- Both sides of an inequality can be multiplied (or divided) by the same positive number without affecting the sign of inequality.
  e.g: $x+y<7$ is same as $(x+y)\times 3<7\times 3$
- When both sides are multiplied or divided by a – ve number, then sign of inequality is reversed.
  e.g: $x+y<7$ is same as $(x+7)\times(-3)>7\times(-3)$

Graphical Solution of Linear Inequalities in Two Variables

- A line divides the cartesian plane into two equal parts. A point in cartesian plane will either lie on a line or will lie in either of half planes. The region containing all the solutions of inequality is called the solution region.
- In order to identify the half plane reprsented by an inequality, it is just sufficient to take any point (a, b) (not on line) and check whether it satisfies the inequality or not. If it satisifes, then inequality represents the half plane and shade the region, which contain the point otherwise the inequality repersents the half plane which does not contain the point within it.

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 7

Permutations & Combinations

Fundamental Principle of Counting (F.P.C.)

F.P.C. of Multiplication: If an event can occur in m different ways, following which another event in n different ways, then the total number of occurance of the events in the given order is $m \times n$

F.P.C. of Addition: If there are two events such that they can performed independently in m and n ways respectively, then either of the two events can be performed in $(m+n)$ ways.

Permutation

Each of the different arrangements which can be made by taking some or all of a number of distinct objects is called a **permutation**.

The no. of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ${}^nP_r$ and is given by

$${}^nP_r = \frac{n!}{(n-r)!}$$

Permutations under given Conditions

The no. of all permutations of 'n' different objects taken r at a time:

(i) When a particular object is to be always included in each arrangement is ${}^{n-1}P_r$

(ii) When a particular object is never taken in each arrangement in ${}^{n-1}P_r$

(iii) The no. of permutation of n different things taken r at a time, where repetition is allowed is $(n)^r$

Permutations of alike Objects

The no. of permutations of n objects taken all at a time, where p_1 objects are of first kind, p_2 objects are of the second kind, ...p_k objects are of the k^{th} kind and rest, if any one are all different is $\frac{n!}{p_1!\,p_2!\,p_3!\ldots p_k!}$

Circular Permutations

1. The no. of circular permutations of 'n' distinct objects is $(n-1)!$
2. If anti-clockwise and clockwise order of arrangements are not distinct then the no. of circular permutations of n distinct items is $\frac{1}{2}(n-1)!$

e.g.: arrangements of beads in a necklace, arrangements of flower in a garland etc.

Some Special Results of ${}^nC_r$

Since,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

- In particular, $r = n$
- ${}^nC_r = \frac{n!}{n!0!} = 1$
- ${}^nC_{n-r} = {}^nC_r$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Relation between ${}^nP_r$ and ${}^nC_r$

${}^nP_r = {}^nC_r \times r!$ $\quad \because 0 < r \leq n$

i.e., Corresponding to each combination of ${}^nC_r$, we have r! permutations, because r objects in every combination can be rearranged in r! ways

Combinations under Certain Conditions

- The no. of arrangements of n distinct objects taken r at a time, so that k particular objects are
  (i) Always included: ${}^{n-k}C_{r-k}.r!$
  (ii) Never included: ${}^{n-k}C_r.r!$
- The no. of selections from 'n' different objects, taking at least one $= {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots\ldots {}^nC_n = 2^n - 1$

Combinations

Each of different selections made by taking source or all of a number of distinct objects or item, irrespective of their arrangements or order in which they are placed, is called a combination.

The no. of combinations of n different things taken r at a time, denoted by ${}^nC_r$ is given by

$${}^nC_r = \frac{n!}{r!(n-r)!};\ 0 \leq r \leq n$$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 8

Binomial Theorem

Binomial Theorem for any Index

If n is negative integer, then n! is not defined. We state binomial theorem in another form

$$(a+b)^n = a^n + \frac{n}{1!}a^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + --- + \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}a^{n-r}b^r + \cdots\cdots + b^n$$

Here, $T_{r+1} = \frac{n(n-1)(n-2)\cdots\cdots(n-r+1)}{r!}a^{n-r}b^r$

General & Middle Terms in Expansion of $(a+b)^n$

The general term of an expansion $(a+b)^n$ is

$T_{r+1} = {}^nC_r\, a^{n-r} b^r,\ 0 \le r \le n, r \in N$

Middle Terms:

1. In $(a+b)^n$, if n is even, then the no. of terms in the expansion is odd. Therefore, there is only one middle term and it is $\left(\frac{n+2}{2}\right)^{th}$ term.

2. In $(a+b)^n$, if n is odd then the no. of terms in the expansion is even. Therefore, there are two middle terms and those are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms.

Some Special Cases $(a+b)^n$

In the expansion of $(a+b)^n$,

(i) Taking a=x and b=−y, we obtain

$(x-y)^n = {}^nC_0x^n - {}^nC_1x^{n-1}y + {}^nC_2\,x^{n-2}y^2 + --- + (-1)^n\, {}^nC_n\, y^n$

(ii) Taking a=1, b=x, we obtain

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \text{-------} + {}^nC_nx^n$

(iii) Taking a=1, b=−x, we obtain

$(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 + \text{----} + (-1)^n\, {}^nC_nx^n$

Definition

If a, b∈R and n∈N, then

$$(a+b)^n = {}^nC_0\,a^nb^0 + {}^nC_1\,a^{n-1}b^1 + {}^nC_2\,a^{n-2}b^2 + \text{------} + {}^nC_n\,a^0b^n$$

- **Remarks:** If the index of the binomial is n then the expansion contains n+1 terms.
- In each term, the sum of indices of a and b is always n.
- Coefficients of the terms in binomial expansion equidistant from both the ends are equal.

$$(a-b)^n = {}^nC_0\,a^nb^0 - {}^nC_1a^{n-1}b^1 + {}^nC_2\,a^{n-2}b^2 + \text{------} + (-1)^n\,{}^nC_na^0b^n$$

Binomial Coefficients

The coefficient ${}^nC_0, {}^nC_1, {}^nC_2, \text{------}\, {}^nC_n$, in the expansion of $(a+b)^n$ are called binomial coefficients and denoted by $C_0, C_1, C_2 \text{----} C_x$, respectively

Properties of binomial coefficients:

(i) $C_0 + C_1 + C_2 + \text{------} + C_n = 2^n$

(ii) $C_0 - C_1 + C_2 - \text{------} + (-1)^n C_n = 0$

(iii) $C_0 + C_2 + C_4 + \text{-----} = C_1 + C_3 + C_5 + \text{----} = 2^{n-1}$

(iv) ${}^nC_{r_1} = {}^nC_{r_2} \Rightarrow r_1 = r_2 \text{ or } r_1 + r_2 = n$

(v) ${}^nC_{r_1} + {}^nC_{r-1} = {}^{1+n}C_r$

(vi) ${}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 9

Sequences and Series

Definitions

Sequence is a function whose domain is the set of N natural numbers.

Real sequence: A sequence whose range is a subset of R is called a real sequence

Series: If $a_1, a_2, a_3, ...a_n$, is a sequence, then the expression $a_1+a_2+a_3+...a_n+..$ is a series. A series is finite or infinite according to the no. of terms in the corresponding sequence as finite or infinite.

Progression: Those sequences whose terms follow certain patterns are called progression.

Arithmetic Progression (A.P.)

An A.P. is a sequence in which terms increase or decrease regularly by the fixed number (same constant). This fixed number is called 'common difference' of the A.P. If 'a' is the first term & 'd' is the common difference and 'l' is the last term of A.P., then general term or the n[th] term of the A.P. is given by $a_n = a + (n-1)d$.

The sum S_n of the first n terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+l]$$

Arithmetic Mean (AM)

If three numbers are in A.P., then the middle term is called AM between the other two, so if a, b, c, are in A.P., b is AM of a and c.

AM for any 'n' +ve numbers $a_1, a_2, a_3, ..., a_n$ is

$$AM = \frac{a_1+a_2+a_3+...+a_n}{n}$$

n-Arithmetic Mean between Two Numbers: If a, b are any two given numbers & a, $A_1, A_2, .., A_n$, b are in A.P. then $A_1, A_2, ..., A_n$ are n AM's between a & b.

$$A_1 = a+\frac{b-a}{n+1}, A_2 = a+\frac{2(b-a)}{n+1} ... A_n = a+\frac{n(b-a)}{n+1}$$

or $A_1 = a+d$, $A_2 = a+2d$, ..., $A_n = a+nd$, where $d = \frac{b-a}{n+1}$

Sum upto terms of some Special Sequences

- Sum of first 'n' natural numbers

$$\sum_{k=1}^{n} k = 1+2+3+...+n = \frac{n(n+1)}{2}$$

- Sum of squares of first n natural numbers

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+...+n^2 = \frac{n(n+1)(2n+1)}{6}$$

- Sum of cubes of first n natural numbers

$$\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+...+n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum_{k=1}^{n} k\right)^2$$

- Sum of first 'n' odd natural numbers

$$\sum_{k=1}^{n}(2k-1) = 1+3+5+...+(2n-1) = n^2$$

Geometric Progression (G.P.)

A sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.This constant factor is called **'Common ratio'**. Usually we denote the first term of a G.P. by 'a' and its common ratio by 'r'. The gereral or n[th] term of G.P. is given by $a_n = ar^{n-1}$ The sum S_n of the first n terms of G.P. is given by

$$S_n = \frac{a(r^n-1)}{r-1} \text{ or } \frac{a(1-r^n)}{1-r} \text{ if } r \neq 1$$

Sum of infinite terms of G.P. is given by

$$S_n = \frac{a}{1-r}; |r|<1$$

Geometric Mean (GM)

- If a, b, c are in G.P., b is the GM between a & c, $b^2 = ac$, therefore $b = \sqrt{ac}$; a > 0, c > 0
- If a, b are two given numbers & a, $G_1, G_2, ..., G_n$, b are in G.P., then $G_1, G_2, G_3 ... G_n$ are n GMs between a & b.

$$G_1 = a\cdot\left(b/a\right)^{1/n+1} G_2 = a\cdot\left(b/a\right)^{2/n+1} ...... G_n = a\cdot\left(b/a\right)^{n/n+1}$$

or

$$G_1 = ar, G_2 = ar^2, ... G_n = ar^{n-1}$$

where $r = \left(b/a\right)^{1/n+1}$,

Arithmetico Geometric Series

In a series,where each term is formed by multiplying the corresponding term of an AP & GP is called Arithmetico-Geometric Series.

Eg: $1+3x+5x^2+7x^3+...$

Here, 1, 3, 5, ... are in AP and 1, x, x^2, ... are in GP.

Properties of AM & GM between two quantities

- If A and G are respectively arithmetic and geometric means between two positive numbers a and b; then the quadratic equation have a, b as it's root is $x^2-2Ax+G^2=0$
- If A and G be the AM & GM between two positive numbers, then the numbers are $A \pm \sqrt{A^2-G^2}$

Relationship between AM and GM

Let A and G be the AM and GM of two given positive real numbers a & b, respectively. Then,

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}$$

Thus, we have , $A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$

$$= \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2} \geq 0 \text{ .... (1)}$$

From (1), we obtain the relationship A ≥ G

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 10

Straight Line

Slope Formula

- If θ is the angle at which a straight line is inclined to be +ve direction of x-axis and $0° \le \theta < 180°$, $\theta \neq 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m doesn't exist, but the line is parallel to y-axis. If $\theta = 0°$, then $m = 0$ and the line is parallel to x-axis.
- If $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_1 \neq x_2$ are points on straight line, then the slope m of the line is given by $m = (y_1 - y_2 / x_1 - x_2)$

Area of Triangle

Area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}\left|x_1(y_1 - y_2) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

Distance Formula

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Section Formula

The $P(x, y)$ divided the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio m:n, then $x = \frac{mx_2 + nx_1}{m + n}$; $y_1 = \frac{my_2 + ny_1}{m + n}$

Note:
- If m/n is +ve, the division is internal, but if m/n is –ve, the division is external.
- If m = n, then P is the mid-point of the line segment joining A & B.

Equation of Straight Line in various forms

1. **POINT-SLOPE FORM**: $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is 'm' and passes through the point (x_1, y_1).
2. **SLOPE INTERCEPT FORM**: $y = mx + c$ is the equation of a straight line whose slope is 'm' and makes an intercept c on the y-axis.
3. **TWO POINT FORM**: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through (x_1, y_1) & (x_2, y_2).
4. **INTERCEPT FORM**: $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on x and y axis respectively.
5. **NORMAL / PERPENDICULAR FORM**: $x\cos\alpha + y\sin\alpha = p$ (where $p > 0, \le 0\ \alpha < 2\pi$) is the equation of a straight line where the length of the perpendicular from origin O on the line is p and this perpendicular makes an angle α with +ve x-axis.
6. **GENERAL FORM**: $ax + by + c = 0$ is the equation of a straight line in general form. In this case, slope of line $= -\frac{a}{b}$

Length of the perpendicular from a point on a line

The length of the perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is:

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Reflection & foot of perpendicular a point about a line

1. The image of a point(x_1, y_1) about a line $ax + by + c = 0$ is:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

2. Similarly, foot of perpendicular from a point on the line is:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{\sqrt{a^2 + b^2}}$$

Perpendicular lines

1. When two lines of the slope m_1 & m_2 are at right angles, the Product of their slope is -1, i.e., $m_1 m_2 = -1$. Thus, any line perpendicular to $y = mx + c$ is of the form, $y = \frac{-1}{m}x + d$ where d is any parameter.
2. Two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are perpendicular if $aa' + bb' = 0$. Thus, any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

Parallel lines

1. When two lines are parallel their slopes are equal. Thus, any line parallel to $y = mx + c$ is of the type $y = mx + d$, where d is any parameter
2. Two lines $ax + by + c = 0$ and $a'x + b'x + c' = 0$ are parallel if $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$
3. The distance between two parallel lines with equations $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

Angle between two straight lines in terms of their slopes

If m_1 and m_2 are the slopes of two intersecting lines ($m_1 m_2 \neq -1$) and θ be the acute angle between them then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 11

Conic Sections

Ellipse

- An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is constant.
- The two fixed points are called the 'foci' of the ellipse.
- The midpoint of line segment joining foci is called the 'centre' of the ellipse.
- The line segment through the foci of the ellipse is called' major axis'.
- The line segment through centre & perpendicular to major axis is called minor axis.
- The end point of the major axis are called the vertices of the ellipse.
- The equation of ellipse with 'foci' on the x-axis is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
- Length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$
- The eccentricity of an ellipse is the ratio of distances from centre of ellipse to one of foci and to one of the vertices of ellipse i.e., $e = c/a$

Parabola

- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane. Fixed line is called 'directrix' of parabola. Fixed point F is called the 'focus'. A line through focus & perpendicular to directrix is called 'axis'. Point of intersection of parabola with axis is called 'vertex'.
- The equation of parabola with focus at (a, 0), a > 0 and directrix x= –a is $y^2=4ax$, where 4a is the length of the latus rectum

Eg: Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).

Sol: Since, vertex is at (0, 0) and focus is at (0, 2) which lies on y-axis, the y-axis is the axis of parabola. Therefore, equation of the parabola is of the form $x^2 = 4ay$. Thus we have $x^2 = 4(2)y$

i.e., $x^2 = 8y$

Definition

Circles, ellipses, parabolas and hyperbolas are known as conic sections because they can be obtained as intersections of plane with a double napped right circular cone.

Hyperbola

- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
- The equation of a hyperbola with foci on the x-axis is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
- The two fixed points are called the 'foci' of the hyperbola.
- The mid-point of the line segment joining the foci is called the 'centre' of the hyperbola.
- The line through the foci is called 'transverse axis'.
- Line through centre and perpendicular to transverse axis is called 'conjugate axis'.
- Points at which hyperbola intersects transverse axis are called 'vertices'.
- Length of the latus rectum of the hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$
- The eccentricity of a hyperbola is the ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of the hyperbola.

Circle

A circle is a set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is called the 'centre' of the circle and the distance from the centre to a point on the circle is called the 'radius' of the circle.

The equation of a circle with centre (h, k) and the radius r is

$$(x-h)^2+(y-k)^2=r^2$$

Eg: Find the equation of the circle with centre (–3, 2) and radius 4.

Sol: Here, h = – 3, k = 2 and r = 4

Therefore, the equation of the required circle is $(x+3)^2+(y-2)^2=16$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 12

Introduction to Three Dimensional Geometry

Coordinates of the Centroid of a Triangle

The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Eg: The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, –6), respectively, find the coordinates of the point C.

Sol: Let the coordinates of C be (x, y, z) and the coordinates of the centroid G be (1, 1, 1). Then $\frac{x+3-1}{3} = 1$, i.e., x=1;

$\frac{y-5+7}{3} = 1$, i.e., $y=1$;

$\frac{z+7-6}{3} = 1$, i.e., z=2. So, C (x,y,z) = (1,1,2)

Coordinates of a Midpoint

The coordinates of the midpoint of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

Eg: Find the midpoint of the line joining two points P(1, –3, 4) and Q(–4, 1, 2).

Sol: Coordinates of the midpoint of the line joining the points P & Q are

$$\left(\frac{1-4}{2}, \frac{-3+1}{2}, \frac{4+2}{2}\right) \text{ i.e. } \left(\frac{-3}{2}, -1, 3\right)$$

Introduction

- In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are three mutually perpendicular lines. The axes are called x, y and z axes.
- The three planes determined by the pair of axes are the coordinate planes, called xy, yz and zx-planes.
- The three coordinate planes divide the space into eight parts known as octants.
- The coordinates of a point P in 3D Geometry is always written in the form of triplet like (x,y,z). Here, x, y and z are the distances from yz, zx and *yx* planes, respectively.

Eg:

- Any point on x-axis is : (x, 0, 0)
- Any point on y-axis is : (0, y, 0)
- Any point on z-axis is : (0, 0, z)

Distance between Two Points

Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Eg: Find the distance between the points P(1, –3, 4) and (–4, 1, 2).

Sol: The distance PQ between the points P & Q is given by

$$PQ = \sqrt{(-4-1)^2 + (1+3)^2 + (2-4)^2}$$

$$= \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

Section Formula

The coordinates of the point R which divides the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally and externally in the ratio m : n are given by

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right) \;\&\; \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$$

respectively.

Eg: Find the coordinates of the point which divides the line segment joining the points (1,–2, 3)and (3, 4, –5) in the ratio 2:3 internally.

Sol : Let P(x, y, z) be the point which divides line segment joining A (1,–2, 3) and B (3, 4, –5) internally in the ratio 2:3. Therefore,

$$x = \frac{2(3)+3(1)}{2+3} = \frac{9}{5} \quad y = \frac{2(4)+3(-2)}{2+3} = \frac{2}{5} \quad z = \frac{2(-5)+3(3)}{2+3} = \frac{-1}{5}$$

Thus, the required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$.

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 13

Limits and Derivatives

Limit

- We say $\lim_{x \to a^-} f(x)$ is the expected value of f at $x=a$ given that the values of f near x to the left of a. This value is called the left hand limit of f at 'a'
- We say $\lim_{x \to a^+} f(x)$ is the expected value of f at $x=a$ given that the values of f near x to the right of a. This value is called the right hand limit of f 'at a'.
- If the right and left hand limits coincide, we call that common value as the limit of $f(x)$ at $x=a$ and denoted it by $\lim_{x \to a} f(x)$.

Eg: Find limit of function $f(x)=(x-1)^2$ at $x=1$.

Sol: For $f(x)=(x-1)^2$

Left hand limit (LHL) (at x=1) $= \lim_{x \to 1^-} (x-1)^2 = 0$

and Right hand limit (RHL) (at x=1) $= \lim_{x \to 1^+} (x-1)^2 = 0$

$\therefore$ LHL = RHL

$\lim_{x \to 1} (x-1)^2 = 0$

Algebra of Limits

For functions f and g the following holds:

- $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
- $\lim_{x \to a} [f(x).g(x)] = \lim_{x \to a} f(x).\lim_{x \to a} g(x)$
- $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\therefore \lim_{x \to a} g(x) \neq 0$

Some Standard Limits

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Some Standard Derivatives

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$

Algebra of derivatives of functions

For functions u and v the following holds:

- $(u \pm v)' = u' \pm v'$
- $(uv)' = u'v + v'u$
- $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ provided all are defined and v ≠ 0

Here, $u' = \frac{du}{dx}$ and $v' = \frac{dv}{dx}$

Derivative of polynomial function

Let f(x) = $a_n x^n + a_{n-1} x^{n-1} + - - + a_1 x + a_0$ be a polynomial function, where a_i s are all real numbers and $a_n \neq 0$. Then the derivative function is given by

$$\frac{d f(x)}{dx} = na_n x^{n-1} + (n-1)a_{n-1} x^{n-2} + - - + 2a_2 x + a_1$$

Derivative of a function at 'a'

The derivative of a function f at a is defined by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Eg: Find derivative of $f(x) = \frac{1}{x}$.

Sol: We have $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{1}{h}\left[\frac{-h}{x(x+h)}\right] = \frac{-1}{x^2}$$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 14

Mathematical Reasoning

Quantifiers

In this statement, the two important symbols are used.
- The symbol '$\forall$' stand for "all values of";
- The symbol '$\exists$' stand for "there exists"

Eg: For every prime number P, P is an irrational number.
- This means that if S denotes the set of all prime numbers, then for all the members of P of the set S, P is an irrational number.

Compound Statement

Many mathematical statements are obtained by combining one or more statements using some connecting words like "and", "or" etc. Each statement is called a "Compound Statement."

Eg:
'The sky is blue' and 'the grass is green' is a compound statement where connecting word is "and";
the components of Compound Statement are
p : The sky is blue
q : The grass is green

Sentence

A sentence is called a mathematically acceptable statement if it is either true or false but not both.

Eg:
- The sum of two positive numbers is positive.
- All prime numbers are odd numbers.
- In above statements, first is 'true' and second is 'false'.

Negation

A statement which is formed by changing the true value of a given statement by using the word like 'no', 'not' is called negation of given statement.
- If P is a statement, then negation is denoted by ~ p

Eg: New Delhi is a city.
The negation of this statement:
→ It is not the case that New Delhi is a city.
→ It is false that New Delhi is a city.
→ New Delhi is not a city.

Contrapositive and Converse Statements

- The contrapositive of a statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- The converse of a statement $p \Rightarrow q$ is the statement $q \Rightarrow p$

Eg: If the physical environment changes, then biological environmental changes.
- **Contrapositive :** If the biological environment does not change then the physical environment does not change.
- **Converse:** If the biological environment changes then physical environment changes.

Validity of Statements

The following methods are used to check the validity of statements
(i) Direct Method
(ii) Contrapositive Method
(iii) Method of Contradiction
(iv) Using a Counter example

Implications

- These are statements with word "if then", "only if" and "if and only if".

Eg:
- r: if a number is a multiple of 9, then it is multiple of 3.
- p: a number is a multiple of 9.
- q: a number is a multiple of 3.
- Then, if p then q is the same as following:

→ p implies q is denoted by $p \Rightarrow q$, then symbol $\Rightarrow$ stands for implies,
→ p is a sufficient condition for q. then symbol $\Rightarrow$
→ p only if q
→ q is a necessary condition for p
→ ~q implies ~p

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 15

Statistics

Variance & Standard Deviation of a discrete frequency distribution

Variance of a discrete frequency distribution

$$\text{Var}(\sigma^2) = \frac{1}{N}\sum f_i(x_i - \bar{x})^2$$

Standard Deviation (S.D.) of a discrete frequency distribution

$$\text{S.D}(\sigma) = \sqrt{\frac{1}{N}\sum f_i(x_i - \bar{x})^2}$$

where $N = \sum f_i$ and $\bar{x}$ = mean

Variance and Standard Deviation for ungrouped data

Variance of ungrouped data

$$\text{Var}(\sigma^2) = \frac{1}{n}\sum (x_i - \bar{x})^2$$

Standard Deviation (S.D.) for un grouped data:

$$\text{S.D.}(\sigma) = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2}$$

Coefficient of Variation

Coefficient of variation (C.V.)

$$= \frac{\sigma}{x} \times 100 \; ; \; \bar{x} \neq 0$$

where $\bar{x}$ is mean

Measures of Dispersion

The degree to which numerical data tend to spread about an average value is called the dispersion of the data. The four dispersions are: Range, Mean deviation, Standard deviation and Variance.

Range

The difference between the highest and the lowest element of a data called its range.

$$range = x_{max} - x_{min}$$

Eg: Find the range of the given data:

4, 7, 8, 9, 10, 12, 13, 17

Here, $x_{max} = 17$ and $x_{min} = 4$

Range = 17–4 = 13

Mean Deviation

Mean Deviation (M.D.) for ungrouped data

- M.D. about mean i.e., $M.D.(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}$, where $\bar{x}$ is mean and n= no. of terms
- M.D. about median i.e., $M.D.(M) = \frac{\sum |x_i - M|}{n}$, where M is median and n= no. of terms

Mean Deviation (M.D.) for grouped data

M.D. about mean i.e., $M.D.(\bar{x}) = \frac{\sum f_i|x_i - \bar{x}|}{n}$

M.D. about median i.e., $M.D.(M) = \frac{1}{N}\sum f_i|x_i - M|$

where $N = \sum f_i$

Median class

Median class is the class interval whose C.F. is just greater than or equal to $\frac{N}{2}$, f, h and C are respectively the lower limit, the frequency, width of the median class and C the cumulative frequency (C.F.) of class just preceding to median class.

Variance & Standard Deviation of a Continuous frequency distribution

Variance of a continuous frequency distribution

$$\text{Var}(\sigma^2) = \frac{1}{N}\Sigma f_i(x_i - \bar{x})^2$$

Standard deviation of a continuous frequency distribution

$$\text{S.D}(\sigma) = \frac{1}{N}\sqrt{N\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2}$$

Step Deviation or Shortcut Method

- $M.D.(\bar{x}) = \frac{1}{N}\sum f_i|x_i - \bar{x}|$

where $\bar{x} = a + \frac{\sum_{i=1}^{n} f_i d_i}{N} \times h$

and $d_i = \frac{x_i - a}{h}$

Here, a = assumed mean

h = common factor

N = sum of frequencies

- $M.D.(M) = \frac{1}{N}\sum f_i|x_i - M|$

where, M(median) $= l + \frac{\frac{N}{2} - C}{f} \times h$

Shortcut method to find variance and standard deviation

Variance

$$(\sigma^2) = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

Standard Deviation

$$\sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}$$

where $y_i = \frac{x_i - A}{h}$

$N = \sum f_i$

A = assumed mean

h = width of class interval

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 16

Probability

Random Experiment

An Experiment is called random experiment if it satisfies the following two conditions:

- It has more than one possible outcome.
- It is not possible to predict the outcome in advance.

Outcome: A possible result of a random experiment is called its outcome.

Sample Space: Set of all possible outcomes of a random experiment is called sample space. It is denoted by symbol 'S'.

Eg: In a toss of a coin, sample space is Head & Tail. i.e., S= {H,T}

Sample Point: Each element of the Sample Space is called a sample point.

Eg: In a toss of a coin, head is a sample point

Equally Likely Outcomes: All outcome with equal probability.

Definition

Probability is the measure of uncertainty of various phenomenon, numerically. It can have positive value from 0 to 1.

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$$

Eg: Probability of getting an even no. in a throw of a die.

Sol. Here, favourable outcomes = {2, 4, 6}

Total no. of outcomes = {1, 2, 3, 4, 5, 6}

$$\text{Probability} = \frac{3}{6} = \frac{1}{2}$$

Event

It is the set of favourable outcomes. Any subset E of a sample space S is called an event.

Eg: Event of getting an even number (outcome) in a throw of a die.

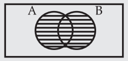
Occurance of event: The event E of a sample space 'S' is said to have occurred if the outcome w of the experiment is such that w ∈E. If the outcome w is such that w ∉E, we say that event E has not occurred.

Types of Events

- **Impossible and Sure Event:** The empty set ϕ is called an Impossible event, where as the whole sample space 'S' is called 'Sure event'.
  Eg: In a rolling of a die, impossible event is that number more than 6 and event of getting number less than or equal to 6 is sure event.
- **Simple Event:** If an event has only one sample point of a sample space, it is called a 'simple event'.
  Eg: In rolling of a die, simple event could be the event of getting number 4.
- **Compound Event:** If an event has more than one sample point, it is called a 'compound event'.
  Eg: In rolling of a die, compound event could be event of getting an even number.
- **Complementary Event:** Complement event to A = 'not A'
  Eg: If an event A= Event of getting odd number in a throw of a die i.e., {1, 3, 5} then , complementary event to A = Event of getting an even number in a throw of a die , i.e. {2, 4, 6}
  A′= {W: W∈ S and W∉A } = S – A (where S is the sample space)

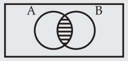
Algebra of Events

- Event A or B or (A∪B)
  A∪B = {w: w∈A or w∈B}



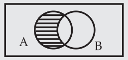
A∪B

- Event A and B or (A∩B)
  A ∩ B = {w: w∈A and w∈B}



A∩B

- Event A but not B or (A–B)
  A–B = A ∩ B′



A–B

Exhaustive Events

Many events that together form sample space are called exhaustive events.

Eg: A die is thrown. Event A = All even outcomes and event B = All odd outcomes. Event A & B together forms exhaustive events as it forms sample space.

Mutually Exclusive Events

Events A & B are called mutually exclusive events if occurance of any one of them excludes occurrance of other event, i.e. they cannot occur simultaneously.

Eg: A die is thrown. Event A= All even outcomes & event B = All odd outcomes. Then A & B are mutually exclusive events, they cannot occur simultaneously.

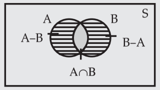
Note: Simple events of a sample space are always mutually exclusive.

Probability of AUB, A∩B and P(notA)

If A and B are any two events, then

- P(AUB) = P(A) + P(B) – P(A∩B)
- P(A∩B) = P(A) + P(B) – P(AUB)

If A and B are mutually exclusive, then P(AUB) = P(A) + P(B)



- Probability of the event 'not A'

P(A′) = P(not A) = 1–P(A)