# Important Questions - 2019-20
## Class - XI (Mathematics)
## Unit - 1 and 2

1. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.
2. By taking an arbitrary element prove that :
   (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (ii) $(A \cap B)' = A' \cup B'$
3. If A, B and C are three sets, then prove that
   $A - (B \cap C) = (A - B) \cup (B - C)$
4. Prove that :
   (i) $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$
   (ii) $A \subseteq B \Longleftrightarrow B' \subseteq A'$
   (iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
5. There are 40 students in a chemistry class & 60 students in a physics class. Find the no of students which are either in physics class or chemistry class in the following cases :
   (i) The two classes meet at the same hour.
   (ii) The two classes meet at different hours and 20 student are enrolled in both the subject.
6. A survey shows that 76% of the Indians like oranges, whereas 62% like bananas what percentage of the indians like both oranges and bananas?
7. Let A and B be two sets such that $n(A) = 20$, $n(A \cup B) = 42$, $n(A \cap B) = 4$, find
   (i) n (B) (ii) n(A–B) (iii) n (B – A)
8. In a survey of 25 students, it was found that 15 had taken maths, 12 had taken physics & 11 had taken chemistry, 5 had taken maths and chemistry, 9 had taken maths and physics, 4 had taken physics and chemistry and 3 had taken all three subjects. Find the number of students that had taken :
   (i) Only Chemistry (ii) only Maths (iii) only Physics (iv) Physics and Chemistry but not Maths
   (v) Maths and Physics but not Chemistry (vi) only one of the subjects
   (vii) atleast one of the three subjects (viii) none of the subjects.
9. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% buy all the three newspaper, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C.
10. In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea, find
    i) how many drink tea and coffee both.
    ii) how many drink coffee but not tea.
11. Find domain and range of following functions :

(i) $f(x) = \dfrac{1}{\sqrt{x-5}}$

(ii) $f(x) = \dfrac{x}{1+x^2}$

(iii) $f(x) = \sqrt{16-x^2}$

(iv) $f(x) = \dfrac{x^2-9}{x-3}$

(v) $f(x) = \dfrac{3}{2-x^2}$

(vi) $f(x) = \dfrac{x^2}{1+x^2}$

(vii) $f(x) = \dfrac{1}{2-\text{Sin } 3x}$

(viii) $f(x) = \dfrac{1}{1-x^2}$

(ix) $f(x) = \dfrac{4-x}{x-4}$

(x) $f(x) = \sqrt{x^2-9}$

12. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

13. If $f(x) = x + \frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

14. If $y = f(x) = \frac{ax - b}{bx - a}$, show that $x = f(y)$

15. If $f(x) = \frac{2x}{1+x^2}$, show that $f(\tan\theta) = \sin 2\theta$

16. If $f(x) = \frac{2^x + 2^{-x}}{2}$, then $f(x+y) \cdot f(x-y)$ is equal to

17. Find the value of:

| | |
|---|---|
| (i) $\sin(3060^0)$ | (v) $\cos(750^0)$ |
| (ii) $\sin(300^0)$ | (vi) $\cot\left(\frac{13\pi}{4}\right)$ |
| (iii) $\tan(240^0)$ | (vii) $\sin(900^0)$ |
| (iv) $\cos(-1440^0)$ | (viii) $\tan\left(-\frac{11\pi}{4}\right)$ |

18. If $\alpha$ and $\beta$ are acute angles such that $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, prove that $\alpha + \beta = \frac{\pi}{4}$

19. Prove that :
(i) $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$
(ii) $\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \frac{1}{\sqrt{2}}\sin A.$

20. If $2\tan\beta + \cot\beta = \tan\alpha$, prove that $\cot\beta = 2\tan(\alpha - \beta)$

21. If $\cos A = \frac{-24}{25}$ and $\cos B = \frac{3}{5}$, where $\pi < A < \frac{3\pi}{2}, \frac{3\pi}{2} < B < 2\pi$ find the following :
(i) $\sin(A+B)$ (ii) $\cos(A+B)$

22. If $\tan A = \frac{3}{4}$, $\cos B = \frac{9}{41}$, where $\pi < A < \frac{3\pi}{2}$ & $0 < B < \frac{\pi}{2}$ find $\tan(A+B)$

23. Find the value of $\tan\pi/8$.

24. If $\tan\alpha = K\tan\beta$, then prove that $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \frac{1+k}{1-k}$

25. Prove that $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$

26. Prove that : $\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x$

27. If $\sin 2A = \lambda\sin 2B$, prove that $\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$

28. Show that : $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = 2\cos\theta$

29. If $\tan^2\theta = 2\tan^2\phi + 1$, prove that $\cos 2\theta + \sin^2\phi = 0$

30. If three angles A,B,C are in A.P., prove that

$$\frac{\text{Sin A} - \text{Sin C}}{\text{Cos C} - \text{Cos A}} = \text{Cot B}$$

31. If $\alpha + \beta = 90^0$, find the maximum and minimum values of $\text{Sin}\alpha\ \text{Sin}\beta$ and $\text{Cos}\alpha\ \text{Cos}\beta$.

32. If $\alpha$ and $\beta$ are the solution of $a\cos\theta + b\sin\theta = C$, then show that :

(i) $\text{Cos}(\alpha+\beta) = \dfrac{a^2 - b^2}{a^2+b^2}$  (ii) $\text{Cos}(\alpha-\beta) = \dfrac{2c^2 - (a^2+b^2)}{a^2+b^2}$

33. If $\text{Sin}\alpha + \text{Sin}\beta = a$ and $\text{Cos}\alpha + \text{Cos}\beta = b$, show that

(i) $\text{Cos}(\alpha+\beta) = \dfrac{b^2 - a^2}{a^2+b^2}$  (ii) $\text{Sin}(\alpha+\beta) = \dfrac{2ab}{a^2+b^2}$

34. If $\alpha$ and $\beta$ are the solution of the equation $a\tan\theta + b\sec\theta = C$, then show that :

$$\tan(\alpha+\beta) = \frac{2ac}{a^2 - c^2}$$

35. If $a\tan\alpha + b\tan\beta = (a+b)\tan\left(\dfrac{\alpha+\beta}{2}\right)$, where $\alpha \neq \beta$, prove that $a\,\text{Cos}\,\beta = b\cos\alpha$

36. If $\text{Cos}(\alpha - \beta) + \text{Cos}(\beta - \gamma) + \text{Cos}(\gamma - \alpha) = -3/2$ prove that

$\cos\alpha + \text{Cos}\,\beta + \text{Cos}\,\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$

37. Prove that $\dfrac{\text{Sin}(B - C)}{\text{Cos B Cos C}} + \dfrac{\text{Sin}(C - A)}{\text{Cos C Cos A}} + \dfrac{\text{Sin}(A - B)}{\text{Cos A Cos B}} = 0$

38. Prove that :

$\tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A.$

39. Find general solution of the following

1) $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

2) $2\,\text{Sin}^2 x + \text{Sin}^2 2x = 2$

3) $\text{Sin}\,\theta + \text{Sin}\,2\theta + \text{Sin}\,3\theta + \text{Sin}\,4\theta = 0$

4) $\tan m\theta + \cot n\theta = 0$

Using P.M.I. Prove that

40. Prove that $n < 2^n$.

41. Prove that $5^n - 5$ is divisible by 4.

42. Prove that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \text{------------} + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

43.4 3 . $\text{Sin}\,\theta + \text{Sin}\,2\theta + \text{------------} + \text{Sin}\,n\theta = \dfrac{\text{Sin}\dfrac{n\theta}{2}\ \text{Sin}\left(\dfrac{n+1}{2}\right)\theta}{\text{Sin}\left(\dfrac{\theta}{2}\right)}$

44. Prove by mathematical Induction that $2.7^n + 3.5^n - 5$ is divisible by 24.

45. $a^{2n} - b^{2n}$ is divisible by a+b

46. $$7 + 77 + 777 + ------------- + \underbrace{777.......7}_{\text{n digit}} = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$$

47. Prove that $(1+x)^n \geq (1 + nx)$ for all natural no. where $x > -1$.

48. Write all subsets of set A = {$\phi$, 1}

49. A and B are two sets such that $A \subset B$. Find the value of $A \cup B$.

50. If A = {1,2,4}, B = {2,4,5} and C = {2,5} then find $(A-B) \times (B-C)$

51. If A = {1,2} & B = {3,4} then how many subsets will $A \times B$ have?

52. In a group of 65 people, 40 like nutrition food, 10 like both nutrition as well as fast food. How many like only fast food? Which type of food is good for health?

53. If A and B are two sets such that $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$, then find $n(B-A)$.

54. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper Z, 9 read both H and Z, 11 read both H and T, 8 read both T and Z and 3 read all three newspapers. Find
i) The no. of people who read atleast one of the newspaper.
ii) The number of people who read exactly one newspaper.

55. Find x and y, if $(x+6, y-2) = (0, 6)$

56. Determine the domain & range of the relation R defined by $R = \{(x, x+5) : x \in 0, 1, 2, 3, 4, 5\}$

57. If A = {x, y, z} & B {1,2}, then find the number of relations from A to B

58. If A = {1, 2, 3} & B = {5, 6}, then show that $A \times B \neq B \times A$

59. If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} of D = {5, 6, 7, 8} then verify that $A \times C \subset B \times D$

60. Define Greatest Integer function.

61. Find the domain of the function $f(x) = -|x - 2|$

62. Find domain of the function : $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2 - 1}}$

63. For any four sets A, B, C & D, prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

64. Let R be the relation on the set z of all integers defined by $R = \{(x, y) : x - y$ is divisible by $n\}$. Prove that
a) $(x, y) \in R \Rightarrow (y, x) \in R$ for all $x, y \in z$
b) $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all $x, y \in z$

65. Solve : $2\,Cos^2x + 3\sin x = 0$

66. If $\cot\alpha \cot\beta = 2$, show that $\dfrac{Cos(\alpha+\beta)}{Cos(\alpha-\beta)} = \dfrac{1}{3}$

67. Solve $\tan^2 x + Cot^2 x = 2$

68. Prove that : $\dfrac{Sec\,8\theta - 1}{Sec\,4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

69. If $2\tan\alpha = 3\tan\beta$, prove that : $\tan(\alpha - \beta) = \dfrac{Sin\,2\beta}{5 - \cos 2\beta}$

70. Prove that $a(CosB + CosC) = 2(b+c)\sin^2 A/2$

71. Prove that $\sum \dfrac{b^2 - c^2}{a^2} \sin 2A = 0$

72. Find the value of Sin $18^0$, Sin $36^0$.

73. In a triangle ABC, prove that $Sin^2\dfrac{A}{2} + Sin^2\dfrac{B}{2} + Sin^2\dfrac{C}{2} = 1 - 2Sin\dfrac{A}{2}Sin\dfrac{B}{2}Sin\dfrac{C}{2}$